Question Number	Scheme	Notes	Marks
9	$P\left(3p^2,\ 6p\right)\quad Q\left(3q^2,\ 6q\right)$		
(a)	$y=\sqrt{12}x^{\frac{1}{2}}$ $\quad \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sqrt{12}}{2}x^{-\frac{1}{2}}=\frac{\sqrt{3}}{\sqrt{3p^2}}=\frac{1}{p}$ $y^2=12x$ $\quad 2y\frac{\mathrm{d}y}{\mathrm{d}x}=12$ $\quad \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6}{y}=\frac{1}{p}$ $y=2at\quad x=at^2$ $\quad \frac{\mathrm{d}y}{\mathrm{d}t}=2a\quad \frac{\mathrm{d}x}{\mathrm{d}t}=2at$ $\quad \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{t}=\frac{1}{p}$ May use $a=3$ and/or $t=p$ Uses calculus to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{p}$. This may be implied by subsequent work.		**B1**
	$y-6p=-p\left(x-3p^2\right)$ or $6p=-p\times 3p^2+c\Rightarrow c=6p+3p^3\Rightarrow y=-px+6p+3p^3$	Forms equation of line with a changed gradient in terms of p and places coordinates correctly	**M1**
	$y+px=6p+3p^3$ *	Correct proof with evidence of calculus. Note that B0M1A1 is not possible.	**A1***
			(3)
(b) Using equation of line PQ	**Note:** There are many variations on the below, e.g. using (3, 0) to form the equation of the line (first M1), then using $\left(3p^2,6p\right)$ or $\left(3q^2,6q\right)$ (second M1)		
	gradient $PQ=\frac{6p-6q}{3p^2-3q^2}=\frac{6(p-q)}{3(p+q)(p-q)}=\frac{2}{p+q}$	A correct expression for the gradient of the chord	**B1**
	e.g. $y-6p=\frac{2}{p+q}\left(x-3p^2\right)$ or $6p=\frac{2}{p+q}\times 3p^2+c\Rightarrow c=6p-\frac{6p^2}{p+q}\Rightarrow y=\frac{2}{p+q}x+6p-\frac{6p^2}{p+q}$	Forms equation of line with their chord gradient in terms of p and q and places coordinates correctly. The example uses $y-y_1=m\left(x-x_1\right)$ with $\left(x_1,y_1\right)=\left(3p^2,6p\right)$, but (3, 0) or $\left(3q^2,6q\right)$ could be used instead.	**M1**
	$-6p=\frac{6-6p^2}{p+q}$	Uses (3, 0) with their line to form an equation in p and q	**M1**
	$p\left(p+q\right)=p^2-1\Rightarrow p^2+pq=p^2-1\Rightarrow pq=-1$*	Fully correct proof with sufficient working	**A1***
	Note: If the gradient is not simplified you may see e.g., $-6p=\frac{6p-6q}{3p^2-3q^2}\left(3-3p^2\right)\Rightarrow -18p^3+18pq^2=18p-18q-18p^3+18p^2q$ $\Rightarrow -pq\left(p-q\right)=p-q\Rightarrow pq=-1$*		
			(4)
Alt 1 Using gradients of PF and QF	grad $PF=\frac{6p}{3p^2-3}$, grad $QF=\frac{6q}{3q^2-3}$	A correct expression for the gradient of PF **or** QF	**B1**
		Correct expressions for gradients of **both** PF **and** QF	**M1**
	$\frac{6p}{3p^2-3}=\frac{6q}{3q^2-3}$	Equates to form an equation in p and q	**M1**
	$pq^2-p=p^2q-q$ $-p+q=p^2q-pq^2$ $-\left(p-q\right)=pq\left(p-q\right)\Rightarrow pq=-1$*	Fully correct proof with sufficient working	**A1***

Alt 2 Similar triangles	$\dfrac{3p^2-3}{3-3q^2}=\dfrac{6p}{-6q}$	Finds **either** the ratio of the widths $\dfrac{3p^2-3}{3-3q^2}$ **or** the heights $\dfrac{6p}{-6q}$	**B1**
		Finds **both** the ratio of the widths $\dfrac{3p^2-3}{3-3q^2}$ **and** the heights $\dfrac{6p}{-6q}$	**M1**
	$\dfrac{p^2-1}{1-q^2}=\dfrac{p}{-q}$	Equates to form an equation in p and q	**M1**
	$-p^2q+q=p-pq^2$ $-pq(p-q)=p-q$ $pq=-1$*	Fully correct proof with sufficient working	**A1***
Alt 3 Using gradients of PQ and PF	gradient $PQ=\dfrac{6p-6q}{3p^2-3q^2}$, gradient $PF=\dfrac{6p}{3p^2-3}$	A correct expression for the gradient of PQ **or** PF	**B1**
		Correct expressions for the gradients of **both** PQ **and** PF	**M1**
	$\dfrac{6p-6q}{3p^2-3q^2}=\dfrac{6p}{3p^2-3}$	Equates to form an equation in p and q	**M1**
	$\Rightarrow\dfrac{1}{p+q}=\dfrac{p}{p^2-1}\Rightarrow p^2-1=p^2+pq\Rightarrow pq=-1$*	Fully correct proof with sufficient working	**A1***
Alt 4 Using lengths: PQ=PF+QF	$PF=3+3p^2$ $QF=3+3q^2$ $PQ=\sqrt{\left(3p^2-3q^2\right)^2+\left(6p-6q\right)^2}$	Correct expression for one of the lengths PQ, PF or QF, or the square of any of these	**B1**
		Attempts all three lengths, or their squares	**M1**
	e.g. $\sqrt{\left(3p^2-3q^2\right)^2+\left(6p-6q\right)^2}=3\left(p^2+q^2\right)+6$	Uses $PQ=PF+QF$ to form an equation in p and q	**M1**
	Completes the proof with no errors $\left(3p^2-3q^2\right)^2+\left(6p-6q\right)^2=\left(3\left(p^2+q^2\right)+6\right)^2$ Multiplying out and dividing both sides by 9 leads to: $p^4-2\left(pq\right)^2+q^4+4p^2-8\left(pq\right)+4q^2=p^4+2\left(pq\right)^2+q^4+4p^2+4q^2+4$ $\Rightarrow\left(pq\right)^2+2\left(pq\right)+1=0\Rightarrow\left(pq+1\right)^2=0\Rightarrow pq=-1$*		**A1***
Alt 5 Sim equations	Line PQ has equation $y=mx+c$ Sub in $\left(3p^2,6p\right),\left(3q^2,6q\right),\left(3,0\right)$: $6p=3mp^2+c$ (1) $6q=3mq^2+c$ (2) $0=3m+c$ (3)	Forms 3 simultaneous equations in p,q,m,c	**B1**
	(1) $-$ (3) $\Rightarrow 6p=3m\left(p^2-1\right)$ (4) (2) $-$ (3) $\Rightarrow 6q=3m\left(q^2-1\right)$ (5)	Eliminates c or m	**M1**
	$\dfrac{6p}{p^2-1}=\dfrac{6q}{q^2-1}$	Eliminates both c and m	**M1**
	$pq^2-p=p^2q-q$ $-p+q=p^2q-pq^2$ $-\left(p-q\right)=pq\left(p-q\right)$	Fully correct proof with sufficient working	**A1***

	$pq=-1$*		
9(c) Way 1	$y+px=6p+3p^3,\quad y+qx=6q+3q^3$ $\Rightarrow 6p+3p^3-px=6q+3q^3-qx$	M1: Eliminates y without sign errors – must form a correct equation.	**M1**
	$6(p-q)+3(p^3-q^3)=(p-q)x$ $x=6+3(p^2+pq+q^2)$ $=3+3(p^2+q^2)$ $=3+3(p+q)^2-6pq$ $=3(p+q)^2+9$	M1: Uses appropriate algebra with sign/coefficient errors only to reach an expression for x of the correct form A1: Correct expression	**M1** **A1**
	$y=6p+3p^3-p\left(3(p+q)^2+9\right)$ $=6p+3p^3-3p^3-6p^2q-3pq^2-9p$ $=-3p-3pq(2p+q)$ $=-3p+6p+3q$ $=3(p+q)$	dM1: Substitutes their expression for x of the correct form to obtain an expression for y of the correct form condoning sign/coefficient errors only A1: Correct expression	**dM1** **A1**
(c) Way 2	$y+px=6p+3p^3,\quad y+qx=6q+3q^3$ $\dfrac{6p+3p^3-y}{p}=\dfrac{6q+3q^3-y}{q}$	M1: Eliminates x without sign errors – must form a correct equation.	**M1**
	$6pq+3p^3q-qy=6pq+3pq^3-pq$ $(p-q)y=3pq(q^2-p^2)$ $(p-q)y=3(p^2-q^2)$ $y=3(p+q)$	M1: Uses appropriate algebra with sign/coefficient errors only to reach an expression for y of the correct form A1: Correct expression	**M1** **A1**
	$x=\dfrac{6p+3p^3-3(p+q)}{p}$ $=6+3p^2+\dfrac{3pq}{p}(p+q)$ $=6+3p^2+3pq+3q^2$ $=9+3(p^2+2pq+q^2)$ $=3(p+q)^2+9$	dM1: Substitutes their expression for y of the correct form to obtain an expression for x of the correct form condoning sign/coefficient errors only A1: Correct expression	**dM1** **A1**
			(5)
(d)	$P(12,12)$ $\Rightarrow p=2,\ q=-\dfrac{1}{2},\ p+q=\dfrac{3}{2}$ $x_R=3\left(\dfrac{3}{2}\right)^2+9,\ y_R=3\left(\dfrac{3}{2}\right)$ $\Rightarrow R\left(\dfrac{63}{4},\dfrac{9}{2}\right)$	M1: Correct method to find one coordinate of R. Substitutes $p=2,\ q=-\dfrac{1}{2}$ or $p+q=\dfrac{3}{2}$ into an expression for x or y from part (c). A1: Correct coordinates of R in simplest form, i.e. $\left(\dfrac{63}{4},\dfrac{9}{2}\right)$ or $\left(15\dfrac{3}{4},4\dfrac{1}{2}\right)$ or $(15.75, 4.5)$ accept $x=...,\ y=...$	**M1** **A1**
			(2)
			Total 14
			PAPER TOTAL: 75