9. The parabola C has equation $y^2 = 12x$

The point $P(3p^2, 6p)$, where $p \neq 0$, lies on C.

(a) Use calculus to show that the normal to C at P has equation

$$y + px = 6p + 3p^3$$

(3)

The point $Q(3q^2, 6q)$, where $q \neq 0$ and $q \neq p$, also lies on C.

Given that the chord PQ passes through the focus of C,

(b) show that $pq = -1$

(4)

The normal to C at P and the normal to C at Q intersect at the point R.

(c) Show that R has coordinates

$$\left(a(p+q)^2 + b,\ c(p+q)\right)$$

where a, b and c are integers to be determined.

(5)

Given that P has coordinates $(12, 12)$

(d) determine, in simplest form, the exact coordinates of R.

(2)

(Total for Question 9 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS