| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **3(a)** | $\frac{2}{(r-1)(r+1)} \equiv \frac{A}{(r-1)} + \frac{B}{(r+1)}$ | | |
| | $\frac{2}{(r-1)(r+1)} \equiv \frac{1}{(r-1)} - \frac{1}{(r+1)}$ | Oe e.g. allow $\frac{1}{(r-1)} + \frac{-1}{(r+1)}$<br>**Must be in terms of *r*.** | B1 |
| | **Do not allow this mark for just finding their constants e.g. *A* = 1, *B* = –1 but allow this mark to be recovered in (b) if the correct partial fractions are seen or used.** | | |
| | | | **(1)** |
| **(b)** | **To score in (b) they must be using partial fractions of the form** $\frac{A}{(r-1)} + \frac{B}{(r+1)}$ | | |
| | $\sum_{r=2}^{n} = \left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \ldots\ldots\ldots + \left(\frac{1}{n-2} - \frac{1}{n}\right) + \left(\frac{1}{n-1} - \frac{1}{n+1}\right)$<br>Attempts at least the first 2 groups of terms and the last 2 groups of terms **which may be implied by their non-cancelling fractions identified below**<br>Allow other letters for *n* (most likely to be *r*) except for the final mark – see below<br>If terms are found beyond the limits of the summation e.g. *r* = 0, *r* = 1, these can be ignored for this mark as long as at least the terms for *r* = 2, 3, *n* – 1 and *n* are seen | | M1 |
| | $\sum_{r=2}^{n} \frac{2}{r^2-1} = 1 + \frac{1}{2} - \frac{1}{n} - \frac{1}{(n+1)}$ | 1, $\frac{1}{2}\left(\text{or}\frac{3}{2}\right)$ identified as the only constant term(s). Follow through their partial fractions so allow $\frac{\textit{their } A}{1}, \frac{\textit{their } A}{2}$ | A1ft **M1 on ePEN** |
| | | $-\frac{1}{n}, -\frac{1}{(n+1)}$ identified as the only algebraic terms. Follow through their partial fractions so allow $\frac{\textit{their } B}{n}, \frac{\textit{their } B}{n+1}$ | A1ft |
| | $= \frac{3n(n+1) - 2(n+1) - 2n}{2n(n+1)}$ | Attempts common denominator from terms of the form $A, \frac{B}{n}, \frac{C}{n+1}$ only.<br>Must see $n(n+1)$ in the denominator and an unsimplified quadratic expression in the numerator.<br>**Dependent on the first method mark.** | **d**M1 |
| | $= \frac{3n^2 - n - 2}{2n(n+1)} = \frac{(3n+2)(n-1)}{2n(n+1)}$ * | Cso. No errors seen but see note below. | A1cso |
| | **Note: If extra terms were considered for the first M mark, (usually those for *r* = 1), allow a full recovery if the correct constant terms are 'extracted'.**<br>**Some candidates attempt** $\sum_{r=1}^{n} \frac{2}{r^2-1} - \sum_{r=1}^{1} \frac{2}{r^2-1}$ **and the same ruling applies as the term for *r* = 1 effectively cancels out, leaving the correct non-cancelling terms.** | | |
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| | | | **(5)** |

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| **3(c)** | $S_{3n}=\sum_{r=2}^{3n}\frac{2}{(r-1)(r+1)}=\frac{(3\times 3n+2)(3n-1)}{2\times 3n(3n+1)}$<br>Correct, possibly unsimplified, expression for $S_{3n}$ using the **given** result in (b) | | B1 |
| | $S_{3n}-S_{n-1}=\frac{(3\times 3n+2)(3n-1)}{2\times 3n(3n+1)}-\frac{(3(n-1)+2)(n-2)}{2(n-1)n}$<br>Attempts $S_{3n}-S_{n-1}$ using the **given** result in (b)<br>If there is any doubt about the "$S_{n-1}$", at least 3 of the $n$'s should be replaced by $n$ - 1 | | M1 |
| | $=\frac{(9n+2)(3n-1)(n-1)-3(3n+1)(3n-1)(n-2)}{6n(3n+1)(n-1)}$<br>$=\frac{(3n-1)(9n^2-7n-2-3(3n^2-5n-2))}{6n(3n+1)(n-1)}$<br>Attempts common denominator involving $n$, $3n + 1$ and $n - 1$ and attempts a factor of $3n - 1$ in the numerator or vice versa. Note that the numerator may be expanded completely to give $24n^2 + 4n - 4$ and the $3n - 1$ then attempted as a factor.<br>**Dependent on the previous mark.** | | **d**M1 |
| | $=\frac{2(3n-1)(2n+1)}{3n(3n+1)(n-1)}$ | Cao | A1 |
| | | | **(4)** |
| | **If (c) is attempted using the method of differences (i.e. repeating the work in (b)) then this scores 0/4 in part (c).** | | |
| | | | **Total 10** |