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7. (a) Show that the substitution $y = vx$ transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + (2 - x^2)y = 2x^3 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2v}{dx^2} - v = 2 \qquad \text{(II)}$$

(5)

(b) By solving differential equation (II), find the general solution of differential equation (I) in the form $y = f(x)$.

(6)

Given that $y = e$ and $\frac{dy}{dx} = e$ at $x = 1$

(c) find the particular solution of differential equation (I).

(4)

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