Question Number	Scheme	Marks
8 (a)	$x = \mathrm{e}^u \quad \dfrac{\mathrm{d}x}{\mathrm{d}u} = \mathrm{e}^u$ or $\dfrac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^{-u}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u} = x$	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}u} \times \dfrac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^{-u}\dfrac{\mathrm{d}y}{\mathrm{d}u}$	M1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\mathrm{e}^{-u}\dfrac{\mathrm{d}u}{\mathrm{d}x}\dfrac{\mathrm{d}y}{\mathrm{d}u} + \mathrm{e}^{-u}\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}\dfrac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^{-2u}\left(-\dfrac{\mathrm{d}y}{\mathrm{d}u} + \dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}\right)$	M1A1
	$x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 3x\dfrac{\mathrm{d}y}{\mathrm{d}x} - 8y = 4\ln x$	
	$\mathrm{e}^{2u} \times \mathrm{e}^{-2u}\left(-\dfrac{\mathrm{d}y}{\mathrm{d}u} + \dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}\right) + 3\mathrm{e}^u \times \mathrm{e}^{-u}\dfrac{\mathrm{d}y}{\mathrm{d}u} - 8y = 4\ln\left(\mathrm{e}^u\right)$	dM1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2} + 2\dfrac{\mathrm{d}y}{\mathrm{d}u} - 8y = 4u$ *	A1*cso (6)

B1	$\dfrac{\mathrm{d}x}{\mathrm{d}u} = \mathrm{e}^u$ oe as shown seen explicitly or used
M1	Obtaining $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ using chain rule here or seen later
M1	Obtaining $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ using product rule (penalise lack of chain rule by the A mark)
A1	Correct expression for $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ any equivalent form
dM1	Substituting in the equation to eliminate x (u and y **only**). Depends on the 2nd M mark
A1*cso	Obtaining the **given** result from completely correct work

	ALTERNATIVE 1	
	$x = \mathrm{e}^u \quad \dfrac{\mathrm{d}x}{\mathrm{d}u} = \mathrm{e}^u = x$	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}u} = \dfrac{\mathrm{d}y}{\mathrm{d}x} \times \dfrac{\mathrm{d}x}{\mathrm{d}u} = x\dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2} = 1\dfrac{\mathrm{d}x}{\mathrm{d}u} \times \dfrac{\mathrm{d}y}{\mathrm{d}x} + x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} \times \dfrac{\mathrm{d}x}{\mathrm{d}u} = x\dfrac{\mathrm{d}y}{\mathrm{d}x} + x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$	M1A1
	$x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{\mathrm{d}^2y}{\mathrm{d}u^2} - \dfrac{\mathrm{d}y}{\mathrm{d}u}$	
	$\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2} - \dfrac{\mathrm{d}y}{\mathrm{d}u}\right) + 3x \times \dfrac{1}{x}\dfrac{\mathrm{d}y}{\mathrm{d}u} - 8y = 4\ln\left(\mathrm{e}^u\right)$	
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2} + 2\dfrac{\mathrm{d}y}{\mathrm{d}u} - 8y = 4u$ *	dM1A1*cso (6)

Question Number	Scheme	Marks
B1	$\frac{dx}{du} = e^u$ oe as shown seen explicitly or used	
M1	Obtaining $\frac{dy}{du}$ using chain rule here or seen later	
M1	Obtaining $\frac{d^2y}{du^2}$ using product rule (penalise lack of chain rule by the A mark)	
A1	Correct expression for $\frac{d^2y}{du^2}$ any equivalent form	
dM1	Substituting in the equation to eliminate x (u and y **only**). Depends on the 2nd M mark	
A1*cso	Obtaining the **given** result from completely correct work	
	ALTERNATIVE 2:	
	$u = \ln x \quad \frac{du}{dx} = \frac{1}{x}$	B1
	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{x}\frac{dy}{du}$	M1
	$\frac{d^2y}{dx^2} = -\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x}\frac{d^2y}{du^2} \times \frac{du}{dx} = -\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x^2}\frac{d^2y}{du^2}$	M1A1
	$x^2\left(-\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x^2}\frac{d^2y}{du^2}\right) + 3x \times \frac{1}{x}\frac{dy}{du} - 8y = 4u$	
	$\frac{d^2y}{du^2} + 2\frac{dy}{du} - 8y = 4u$ *	M1A1*cso
	Notes as for main scheme	

There are also **other solutions** which will appear, either starting from equation II and obtaining equation I, or mixing letters x, y and u until the final stage.
Mark as follows:

B1	as shown in schemes above
M1	obtaining a first derivative with chain rule
M1	obtaining a second derivative with product rule
A1	correct second derivative with 2 or 3 variables present
dM1	Either substitute in equation I or substitute in equation II according to method chosen **and** obtain an equation with only y and u (following sub in eqn I) or with only x and y (following sub in eqn II)
A1cso	Obtaining the required result from completely correct work

Question Number	Scheme	Marks
(b)	$m^2+2m-8=0$	
	$(m+4)(m-2)=0, \quad m=-4,2$	M1A1
	CF $= Ae^{-4u}+Be^{2u}$	A1
	PI: try $y=au+b$ (or $y=cu^2+au+b$ different derivatives, $c=0$)	
	$\frac{dy}{du}=a \quad \frac{d^2y}{du^2}=0$	M1
	$0+2a-8(au+b)=4u$	
	$a=-\frac{1}{2} \quad b=-\frac{1}{8}$	dM1A1
	$\therefore y=Ae^{-4u}+Be^{2u}-\frac{1}{2}u-\frac{1}{8}$	B1ft (7)
(c)	$y=Ax^{-4}+Bx^2-\frac{1}{2}\ln x-\frac{1}{8}$	B1 (1)
		[14]

(b) M1	Writing down the correct aux equation and solving to $m=\ldots$ (usual rules)
A1	Correct solution $(m=-4,2)$
A1	Correct CF – can use any (single) variable
M1	Using an appropriate PI and finding $\frac{dy}{du}$ **and** $\frac{d^2y}{du^2}$ Use of $y=\lambda u$ scores M0
dM1	Substitute in the equation to obtain values for the unknowns. Depends on the second M1
A1	Correct unknowns two or three (with $c=0$)
B1ft	A complete solution, follow through their CF and a non-zero PI. Must have $y=$ a function of u Allow recovery of incorrect variables.
(c) B1	Reverse the substitution to obtain a correct expression for y in terms of x No ft here x^{-4} or $e^{-4\ln x}$ and x^2 or $e^{2\ln x}$ allowed. Must start $y=\ldots$

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