Question	Scheme	Marks
3(a)	Meet when $x^2-8x=\dfrac{4x}{4-x}\Rightarrow\left(x^2-8x\right)\left(4-x\right)=4x\Rightarrow x\left(4x-32-x^2+8x-4\right)=0$	**M1**
	(so $x=0$ or) $x^2-12x+36=0$	**A1**
	$\Rightarrow x\left(x-6\right)^2=0\Rightarrow x=...$	**M1**
	Meet at (6,−12)	**A1**
	e.g. touch at (6,−12) as repeated root.	**B1**
		(5)
Alt	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^2-8x\right)=2x-8$ and $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{4x}{4-x}\right)=\dfrac{4(4-x)-4x(-1)}{(4-x)^2}=\dfrac{16}{(4-x)^2}$	**M1A1**
	$2x-8=\dfrac{16}{(4-x)^2}\Rightarrow(x-4)^3=8\Rightarrow x=...$	**M1**
	Meet at (6,−12)	**A1**
	e.g. $6^2-6\times9=-12$ and $\dfrac{4\times6}{4-6}=-12$, so curves meet at tangent at (6,−12)	**B1**
		(5)
(b)	$x^2-8x=\dfrac{4x}{4+x}\Rightarrow x\left(x-8\right)\left(4+x\right)-4x=0\Rightarrow x\left(x^2-4x-36\right)=0\Rightarrow x=..$	**M1**
	$x=(0),2\pm2\sqrt{10}\Rightarrow$ critical value is (0 and) $2-2\sqrt{10}$	**A1**
	Other C.V.'s are 0, ±4	**B1**
	E.g. extremes are $x<2-2\sqrt{10}$ and $x>6$ or any two suitable ranges.	**M1**
	Solution is $x<2-2\sqrt{10},-4<x<0,4<x<6,x>6$	**A1A1**
		(6)
		(11 marks)

Notes:

(a)

M1: Attempts to find intersection by setting equations equal and cross multiplies and factorises the x out or cancels.

A1: Correct quadratic reached. May be implied by solutions of 0,6 seen from the cubic (by calculator)

M1: Solves the quadratic to find roots.

A1: Obtains the correct point where the curves meet.

B1: Correct reason given for why the curves touch. Accepted "repeated root" as reason. As a minimum, accept "$(x-6)^2=0$ therefore touches". Alternatively, accept discriminant = 0 shown with conclusion, or may find gradient at both points and show equal, with conclusion.

Alt:

M1: Attempts derivatives of both curves

A1: Both derivatives correct.

M1: Sets derivatives equal and solves to find x value where gradients agree.

A1: Obtains the correct point where the curves meet.

B1: Correct value checked in both curves with conclusion that they meet at a tangent or equivalent working as per main scheme.

(b)

M1: Attempts to find the intersection of the other branch of $\frac{4x}{4-|x|}$ with x^2-8x. Allow for any attempt at solving $\frac{4x}{4+x}=x^2-8x$ that reaches a value for x

A1: Correct value of $2-2\sqrt{10}$ identified. (No need to see the second root rejected for this mark.)

B1: Both 0 and ± 4 identified as critical values for the ranges needed at some stage in working.

M1: Forms at least two suitable ranges from their critical values (allow if e.g. $\leqslant$ is used instead of <). Likely to be the extreme ranges, so look for $x <$ their $2-2\sqrt{10}$ and $x >$ their 6. However, allow if this latter is included as part of the range $x > 4$ for this mark.

A1: At least two correct ranges.

A1: Fully correct answer as in scheme.