Question	Scheme	Marks
5(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\left(4+\ln x\right)^{-\frac{1}{2}}\times\frac{1}{x}$	**M1 A1**
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{2}\frac{0-\left(\sqrt{4+\ln x}+x\times\frac{1}{2}\left(4+\ln x\right)^{-\frac{1}{2}}\times\frac{1}{x}\right)}{x^2\left(4+\ln x\right)}$ or $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{1}{4x}\left(4+\ln x\right)^{-\frac{3}{2}}\times\frac{1}{x}-\frac{1}{x^2}\times\frac{1}{2}\left(4+\ln x\right)^{-\frac{1}{2}}$ oe	**M1**
	$=\frac{\ldots}{4x^2\left(4+\ln x\right)^{\frac{3}{2}}}=-\frac{9+2\ln x}{4x^2\left(4+\ln x\right)^{\frac{3}{2}}}$ *	**M1 A1***
		(5)
Alt(a)	$y^2=4+\ln x\Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x}$	**M1 A1**
	$\Rightarrow 2y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=-\frac{1}{x^2}$	**M1**
	$\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{1}{2yx^2}-\frac{2}{8x^2y^3}=\frac{-2y^2-1}{4x^2y^3}$	**M1**
	$=-\frac{9+2\ln x}{4x^2\left(4+\ln x\right)^{\frac{3}{2}}}$ *	**A1***
		(5)
(b)	$y_{x=1}=2,\ \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=1}=\frac{1}{4},\ \left.\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right\|_{x=1}=-\frac{9}{32}$	**M1**
	So $y=2+\frac{1}{4}\left(x-1\right)-\frac{1}{2!}\times\frac{9}{32}\left(x-1\right)^2+\ldots$	**M1**
	$=2+\frac{1}{4}\left(x-1\right)-\frac{9}{64}\left(x-1\right)^2+\ldots$	**A1**
		(3)
		(8 marks)

Notes:

(a)

M1: Attempts the derivative of y using the chain rule, look for $\frac{K}{x}\left(4+\ln x\right)^{-\frac{1}{2}}$ oe

A1: Correct derivative.

M1: Attempts the second derivative of y using the product or quotient rule and chain rule. Look for the correct form for their $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the answer up to slips in coefficients.

M1: Attempts to simplify to get correct denominator. Must be correct work for their second derivative, but may have been errors in differentiating.

A1*: For a correct unsimplified second derivative, with no errors before reaching the given answer.

Note it is a given answer so needs a suitable intermediate line with at least the formation of the correct common denominator between two fractions before reaching the answer.

Alt:

M1: Squares and uses implicit differentiation to achieve $\alpha y\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\beta}{x}$

A1: Correct derivative.

M1: Differentiates again using implicit differentiation and product rule. Look for $\gamma y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\delta\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\frac{\nu}{x^2}$

M1: Makes $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ the subject and forms single fraction with denominator kx^2y^3

A1*: Obtains the correct second derivative, with no errors seen in working.

(b)

M1: Evaluates y, $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at $x = 1$, if substitution is not seen, accept stated values for all three following attempts at the first and second derivatives as an attempt to find these.

M1: Applies Taylor's theorem with their values.

A1: Correct expression (don't be concerned if the $y =$ is missing.)

5(b) Alt	$y=\sqrt{4+\ln\left(1+(x-1)\right)}=\sqrt{4+\left((x-1)-\frac{(x-1)^2}{2}+\ldots\right)}$	**M1**
	$=4^{\frac{1}{2}}+\frac{1}{2}\times4^{-\frac{1}{2}}\times\left((x-1)-\frac{(x-1)^2}{2}\right)+\frac{\frac{1}{2}\times-\frac{1}{2}}{2!}\times4^{-\frac{3}{2}}\times\left((x-1)-\ldots\right)^2+\ldots$	**M1**
	$=2+\frac{1}{4}(x-1)-\frac{1}{8}(x-1)^2-\frac{1}{64}(x-1)^2+\ldots=2+\frac{1}{4}\left(x-1\right)-\frac{9}{64}\left(x-1\right)^2+\ldots$	**A1**
		(3)

Notes:

M1: Writes the x as $1+(x-1)$ and attempts to expand using the Maclaurin series for $\ln(1+x)$ with correct expansion of $\ln(1+(x-1))$.

M1: Attempts a binomial expansion using their ln expansion. Alternatively, may gain this before the first M if they expand using ln's, e.g. $4^{\frac{1}{2}}+\frac{1}{2}4^{-\frac{1}{2}}\ln x+\frac{\frac{1}{2}\times\frac{-1}{2}}{2!}(\ln x)^2$

A1: Fully **c**orrect expression (don't be concerned if the $y =$ is missing.)