Question	Scheme	Marks
6(a)	Let $x = \arctan A$ and $y = \arctan B$ then $\tan(x-y) = \dfrac{\tan x - \tan y}{1+\tan x \tan y}$ Or $\tan(\arctan A - \arctan B) = \dfrac{\tan\arctan A - \tan\arctan B}{1+\tan\arctan A \tan\arctan B}$	**B1**
	$\tan(x-y) = \dfrac{A-B}{1+AB} \Rightarrow x-y = \arctan\left(\dfrac{A-B}{1+AB}\right)$	**M1**
	So $\arctan A - \arctan B = x-y = \arctan\left(\dfrac{A-B}{1+AB}\right)$*	**A1***
		(3)
(b)	$A = r+2, B = r \Rightarrow \left(\dfrac{A-B}{1+AB} =\right) \dfrac{r+2-r}{1+(r+2)r} = \dfrac{2}{\ldots}$	**M1**
	$= \dfrac{2}{r^2+2r+1} = \dfrac{2}{(1+r)^2}$*	**A1***
		(2)
(c)	$\displaystyle\sum_{r=1}^{n} \arctan\left(\frac{2}{(1+r)^2}\right) = \sum_{r=1}^{n} \big(\arctan(r+2) - \arctan(r)\big) = \ldots$	**M1**
	$= \big(\cancel{\arctan 3} - \arctan 1\big) + \big(\cancel{\arctan 4} - \arctan 2\big) + \big(\cancel{\arctan 5} - \cancel{\arctan 3}\big) + \ldots$ $+ \big(\arctan(n+1) - \cancel{\arctan(n-1)}\big) + \big(\arctan(n+2) - \cancel{\arctan n}\big)$	**A1**
	$= \arctan(n+2) + \arctan(n+1) - \arctan 2 - \arctan 1$	**M1**
	$= \arctan(n+2) + \arctan(n+1) - \arctan 2 - \dfrac{\pi}{4}$	**A1**
		(4)
(d)	As $n \to \infty$, $\arctan n \to \dfrac{\pi}{2}$	**M1**
	So $\displaystyle\sum_{r=1}^{\infty} \arctan\left(\frac{2}{(1+r)^2}\right) = \frac{\pi}{2} + \frac{\pi}{2} - \arctan 2 - \frac{\pi}{4} = \frac{3\pi}{4} - \arctan 2$	**A1**
		(2)
		(11 marks)

Notes:

(a)

B1: For any correct statement or use of the compound angle formula with **consistent variables** of x and y or arctan A and arctan B. Can be either way round (may be working in reverse).

M1: Attempts to apply tan or arctan on an appropriate identity with either x and y or arctan A and arctan B.

Should have $\dfrac{\tan x \pm \tan y}{1 \pm \tan x \tan y}$ (oe with arctans or A's and B's) as part of the identity, and allow if they change between x,y and arctan's during the step.

A1*: Must have scored the B and M marks. Replaces tan x and tan y by A and B respectively if appropriate with fully correct work leading to the given result and conclusion made and no erroneous statements.

Note: for working in reverse e.g.

Let $x = \arctan A$ and $y = \arctan B$ then

$$\arctan A - \arctan B = \arctan\left(\frac{A-B}{1+AB}\right) \Leftrightarrow x - y = \arctan\left(\frac{A-B}{1+AB}\right) \Leftrightarrow \tan(x-y) = \frac{A-B}{1+AB}$$ Scores M1

$$\Leftrightarrow \tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \tan y}$$ Scores B1 – but enter as the first mark.

Which is the correct identity for $\tan(x-y)$ hence the result is true. Score A1

The conclusion here must include reference to the identity being true, e.g. with a tick, or statement, before deducing the final result.

(b)

M1: Substitutes in $A = r + 2$ and $B = r$ and simplifies the numerator to 2 (may be implied)

A1*: Expands the denominator (must be seen) and then factorises to the given result, no errors seen.

(c)

M1: Applies the result of (a) to the series – allow if they have a different A and B due to error.

A1: At least first three and final two brackets of terms correctly written out – must be clear enough to show cancelling.

M1: Extracts the non-cancelling terms.

A1: Correct result with no errors seen – must see the arctan 1 before reaching $\dfrac{\pi}{4}$.

Note: Insufficient terms to gain the first A is not an error, so M1A0M1A1 is possible if e.g. only the first two terms are shown. Condone missing brackets on arctan $n + 1$ etc.

(d)

M1: Identifies the value arctan tends towards as n increase. Need not see limits, as long as the value is identified.

A1: Correct answer.