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6. Given that $A > B > 0$, by letting $x = \arctan A$ and $y = \arctan B$

(a) prove that

$$\arctan A - \arctan B = \arctan\left(\frac{A - B}{1 + AB}\right)$$

(3)

(b) Show that when $A = r + 2$ and $B = r$

$$\frac{A - B}{1 + AB} = \frac{2}{(1 + r)^2}$$

(2)

(c) Hence, using the method of differences, show that

$$\sum_{r=1}^{n} \arctan\left(\frac{2}{(1 + r)^2}\right) = \arctan(n + p) + \arctan(n + q) - \arctan 2 - \frac{\pi}{4}$$

where p and q are integers to be determined.

(4)

(d) Hence, making your reasoning clear, determine

$$\sum_{r=1}^{\infty} \arctan\left(\frac{2}{(1 + r)^2}\right)$$

giving the answer in the form $k\pi - \arctan 2$, where k is a constant.

(2)

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