Question	Scheme	Marks
7(a)	$z=(0+)\mathrm{i}y \Rightarrow w=\dfrac{(1+\mathrm{i})\mathrm{i}y+2(1-\mathrm{i})}{\mathrm{i}y-\mathrm{i}}=\dfrac{-y+2+\mathrm{i}(y-2)}{\mathrm{i}(y-1)}=\dfrac{y-2+\mathrm{i}(y-2)}{y-1}$	M1
	$\Rightarrow u=v$ or $\mathrm{Im}\,w=\mathrm{Re}\,w$	A1
		(2)
(b)	$w=\dfrac{(1+\mathrm{i})z+2(1-\mathrm{i})}{z-\mathrm{i}} \Rightarrow z=\dfrac{2(1-\mathrm{i})+\mathrm{i}w}{w-1-\mathrm{i}}=\dfrac{2-v+\mathrm{i}(u-2)}{u-1+\mathrm{i}(v-1)}$	M1
	$\dfrac{2-v+\mathrm{i}(u-2)}{u-1+\mathrm{i}(v-1)}\times\dfrac{u-1-\mathrm{i}(v-1)}{u-1-\mathrm{i}(v-1)}$ $=\dfrac{(2-v)(u-1)+(u-2)(v-1)+\mathrm{i}\big((u-1)(u-2)-(2-v)(v-1)\big)}{\ldots}$ $\mathrm{Im}\,z=0 \Rightarrow (u-1)(u-2)-(2-v)(v-1)=0$	M1
	$\Rightarrow (u-1)(u-2)-(2-v)(v-1)=0 \Rightarrow u^2-3u+2+v^2-3v+2=0$	A1
	$\Rightarrow \left(u-\dfrac{3}{2}\right)^2+\left(v-\dfrac{3}{2}\right)^2=\dfrac{1}{2}$	M1
	Centre is $\dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}$ and radius is $\dfrac{\sqrt{2}}{2}$	A1A1
		(6)
		(8 marks)

Notes:

(a)

M1: Correct method to find the equation of the image line – e.g. substitutes in $z=\mathrm{i}y$ and rearranges to Cartesian form. May use $x+\mathrm{i}y$ and later set $x = 0$. Alternatively, may start as in (b) and then set $(2-v)(u-1)+(u-2)(v-1)=0 \Rightarrow 2u-v-uv-2+uv+2-2v-u=0$ etc.

Another alternative is to find the image points of two points on the imaginary axis and to find the line from these.

A1: For $u = v$ oe equation. Accept Im w = Re w, or $x = y$ if they have set $w = x + \mathrm{i}y$.

(b)

Note: Accept work done in part (a) that is relevant to part (b) for credit if appropriate.

M1: Makes z the subject, substitutes $w = u + \mathrm{i}v$ into the equation.

M1: Multiplies the numerator by the complex conjugate of denominator **and** extracts the imaginary part and sets it equal to zero to form an equation in u and v. Do not be concerned about the denominator.

A1: Correct equation in u and v for the circle.

M1: Completes the square on their equation to extract centre and radius. Not dependent, so allow as long as a suitable equation in u and v has been reached.

A1: Correct centre or correct radius. Accept either $\dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}$ or $\left(\dfrac{3}{2},\dfrac{3}{2}\right)$ for the centre.

A1: Correct centre and correct radius. As above. Accept equivalent forms (need not be simplified)

Allow the final two A marks if all that is wrong is an error in the denominator. (M1M0A0M1A1A1 is possible.)

7(b) **Alt1**	Real axis is $z = x(+0\mathrm{i})$, so $u+\mathrm{i}v = \dfrac{(1+\mathrm{i})x+2(1-\mathrm{i})}{x-\mathrm{i}} = \dfrac{(1+\mathrm{i})x+2(1-\mathrm{i})}{x-\mathrm{i}} \times \dfrac{x+\mathrm{i}}{x+\mathrm{i}} =$ $\dfrac{(1+\mathrm{i})x^2+2x(1-\mathrm{i})+(\mathrm{i}-1)x+2(\mathrm{i}+1)}{x^2+1} = \dfrac{x^2+x+2+\mathrm{i}\left(x^2-x+2\right)}{x^2+1}$	**M1**
	$u = \dfrac{x^2+x+2}{x^2+1} = 1+\dfrac{x+1}{x^2+1}; v = \dfrac{x^2-x+2}{x^2+1} = 1-\dfrac{x-1}{x^2+1} \Rightarrow u+v = 2+\dfrac{2}{x^2+1}$ $\Rightarrow (u-1)^2+(v-1)^2 = \dfrac{(x+1)^2+(x-1)^2}{\left(x^2+1\right)^2} = \dfrac{2x^2+2}{\left(x^2+1\right)^2} = \dfrac{2}{x^2+1} = u+v-2$	**M1** **A1**
	$\Rightarrow \left(u-\dfrac{3}{2}\right)^2+\left(v-\dfrac{3}{2}\right)^2 = \dfrac{1}{2}$	**M1**
	Centre is $\dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}$ and radius is $\dfrac{\sqrt{2}}{2}$	**A1A1**
		(6)

Notes

M1: Sets $z = x$ in the equation (or uses $x + iy$ and later sets $y = 0$) and multiplies by complex conjugate.
M1: Eliminates x from the equations (one suitable method is shown, others are possible).
A1: Correct equation in u and v for the circle.
M1: Completes the square on their equation to extract centre and radius
A1: Correct centre or correct radius. Accept either $\dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}$ or $\left(\dfrac{3}{2},\dfrac{3}{2}\right)$ for the centre.
A1: Correct centre and correct radius. As above.

7(b) **Alt 2**	Unlikely to be seen As i and $-\mathrm{i}$ are inverse points of the line, so their images are inverse points of the circle. $\mathrm{i} \to \infty,\ -\mathrm{i} \to \dfrac{-\mathrm{i}+1+2-2\mathrm{i}}{-2\mathrm{i}} = \dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}$ Hence (as ∞ is the other point) the centre is $\dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}$ $0 \to \dfrac{2-2\mathrm{i}}{-\mathrm{i}} = 2+2\mathrm{i}$ So radius is $\left\lvert\dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}-2-2\mathrm{i}\right\rvert = \ldots$ $= \dfrac{\sqrt{2}}{2}$	**M1** **M1** **A1** **M1** **A1** **A1**
(b) Alt 3	M1: Attempt to find images of three different points on the real axis. M1: Correct method to find centre from three points – e.g. intersection of two perpendicular bisectors. A1: Correct equation for the centre. M1: Uses centre and one point to find radius. A1: Correct centre A1: Correct radius	