Question Number	Scheme	Marks
8 (a)	$(\cos\theta+\mathrm{i}\sin\theta)^5=\cos5\theta+\mathrm{i}\sin5\theta$	B1
	$=\cos^5\theta+5\cos^4\theta(\mathrm{i}\sin\theta)+\frac{5\times4}{2!}\cos^3\theta(\mathrm{i}\sin\theta)^2$ $+\frac{5\times4\times3}{3!}\cos^2\theta(\mathrm{i}\sin\theta)^3+\frac{5\times4\times3\times2}{4!}\cos\theta(\mathrm{i}\sin\theta)^4+(\mathrm{i}\sin\theta)^5$	M1
	$=\cos^5\theta+\underline{5\mathrm{i}\cos^4\theta\sin\theta}-10\cos^3\theta\sin^2\theta\underline{-10\mathrm{i}\cos^2\theta\sin^3\theta}+5\cos\theta\sin^4\theta\underline{+\mathrm{i}\sin^5\theta}$	A1
	$\sin5\theta=5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+\sin^5\theta$ $=5(1-\sin^2\theta)^2\sin\theta-10(1-\sin^2\theta)\sin^3\theta+\sin^5\theta\frac{\mathrm{d}y}{\mathrm{d}x}$ $=5(1-2\sin^2\theta+\sin^4\theta)\sin\theta-10(1-\sin^2\theta)\sin^3\theta+\sin^5\theta$	M1
	$\sin5\theta=16\sin^5\theta-20\sin^3\theta+5\sin\theta$ *	A1* (5)
	Alternative: Using "$z-\frac{1}{z}$" $z^5-\frac{1}{z^5}=2\mathrm{i}\sin5\theta$ oe	B1
	Binomial expansion of $\left(z-\frac{1}{z}\right)^5$	M1
	$32\sin^5\theta=2\sin5\theta-10\sin3\theta+20\sin\theta$	A1
	Uses double angle formulae etc to obtain $\sin3\theta=3\sin\theta-4\sin^3\theta$ and then use it in their expansion	M1
	$\sin5\theta=16\sin^5\theta-20\sin^3\theta+5\sin\theta$ *	A1* (5)
(b)	Let $x=\sin\theta$ $16x^5-20x^3+5x=-\frac{1}{5}\Rightarrow\sin5\theta=...$	M1A1
	$\Rightarrow\theta=\frac{1}{5}\sin^{-1}\left(\pm\frac{1}{5}\right)=38.306\ (\text{or } -2.307, 69.692.\ 110.306,\ 141.693, 182.306)$	dM1
	(or in radians −0.0402…0.6685..., 1.216…,1.925…, 2.473…)	
	Two of (awrt) $x=\sin\theta=-0.963,\ -0.555,\ -0.040,\ 0.620,\ 0.938$	A1
	All of (awrt) $x=\sin\theta=-0.963,\ -0.555,\ -0.040,\ 0.620,\ 0.938$	A1 (5)
(c)	$\int_0^{\frac{\pi}{4}}(4\sin^5\theta-5\sin^3\theta-6\sin\theta)\mathrm{d}\theta=\left(\int_0^{\frac{\pi}{4}}\frac{1}{4}(\sin5\theta-5\sin\theta)-6\sin\theta\right)\mathrm{d}\theta$	M1
	$=\left[\frac{1}{4}\left(-\frac{1}{5}\cos5\theta+5\cos\theta\right)+6\cos\theta\right]_0^{\frac{\pi}{4}}\left(=\left[-\frac{1}{20}\cos5\theta+\frac{29}{4}\cos\theta\right]_0^{\frac{\pi}{4}}\right)$	A1
	$\frac{1}{4}\left[-\frac{1}{5}\cos\frac{5\pi}{4}+5\cos\frac{\pi}{4}-\left(-\frac{1}{5}+5\right)\right]+6\cos\frac{\pi}{4}-6$	
	$=\frac{1}{4}\left[\frac{1}{5}\times\frac{1}{\text{Ö}2}+\frac{5}{\text{Ö}2}-4\frac{4}{5}\right]+\frac{6}{\sqrt{2}}-6$	dM1
	$=\frac{73\sqrt{2}}{20}-\frac{36}{5}$ oe	A1 (4)
		[14]
	Notes	

(a)	
B1	Applies de Moivre correctly. Need not see full statement, but must be correctly applied.
M1	Use binomial theorem to expand $(\cos\theta + i\sin\theta)^5$ May only show imaginary parts - ignore errors in real parts. Binomial coefficients must be evaluated.
A1	Simplify coefficients to obtain a simplified result with all imaginary terms correct
M1	Equate imaginary parts and obtain an expression for $\sin 5\theta$ in terms of powers of $\sin\theta$ No $\cos\theta$ now
A1*	Correct **given** result obtained from fully correct working with at least one intermediate line wit the $(1-\sin^2\theta)^2$ expanded. Must see both sides of answer (may be split across lines). A0 if equating of imaginary terms is not clearly implied.
(b)	
	Note Answers only with no working score no marks as the "hence" has not been used. But if the first M1A1 gained then dM1 may be implied by a correct answer.
M1	Use substitution $x = \sin\theta$ and attempts to use the result from (a) to obtain a value for $\sin 5\theta$
A1	Correct value for $\sin 5\theta$
dM1	Proceeds to apply arcsin and divide by 5 to obtain at least one value for θ. Note for $\sin 5\theta = \frac{1}{5}$ the values you may see are the negatives of the true answers. FYI: $(5\theta = -11.53..., 191.53..., 348.46..., 551.53..., 708.46..., 911.53...)$ (Or in radians $-0.201\ldots$ 3.3428..., 6.0819..., 9.6260..., 12.365..., 15.909…)
A1	Proceeds to take sin and achieve at least 2 different correct values for x or $\sin\theta$
A1	For all 5 values of x or $\sin\theta$ awrt 3 d.p. (allow 0.62 and – 0.04)
(c)	
M1	Use previous work to change the integrand into a function that can be integrated
A1	Correct result after integrating. Any limits shown can be ignored
dM1	Substitute given limits, subtracts and uses exact numerical values for trig functions
A1	Final answer correct (oe provided in the given form)

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