8. (a) Use de Moivre's theorem to show that

$$\sin 5\theta \equiv 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$

(5)

(b) Hence determine the five distinct solutions of the equation

$$16x^5 - 20x^3 + 5x + \frac{1}{5} = 0$$

giving your answers to 3 decimal places.

(5)

(c) Use the identity given in part (a) to show that

$$\int_0^{\frac{\pi}{4}} \left(4\sin^5\theta - 5\sin^3\theta - 6\sin\theta\right) \mathrm{d}\theta = a\sqrt{2} + b$$

where a and b are rational numbers to be determined.

(4)

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(Total for Question 8 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

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