| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **8(a)** | $y = r\sin\theta = (1-\sin\theta)\sin\theta = \sin\theta - \sin^2\theta$<br>$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}\theta} = \cos\theta - 2\sin\theta\cos\theta$<br>or e.g.<br>$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}\theta} = \cos\theta - \sin 2\theta$ | Differentiates $(1-\sin\theta)\sin\theta$ to achieve $\pm\cos\theta \pm k\sin\theta\cos\theta$ or equivalent. Use of $y = r\cos\theta$ or $x = r\cos\theta$ scores M0 | M1 |
| | | Correct derivative in any form. | A1 |
| | $\cos\theta - 2\sin\theta\cos\theta = 0 \Rightarrow \cos\theta(1-2\sin\theta) = 0 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = ...$<br>Solves to find a value for $\theta$. Depends on the first M. | | dM1 |
| | $\left(\frac{1}{2}, \frac{\pi}{6}\right)$<br>Correct coordinates and no others. Isw if necessary e.g. if written as $\left(\frac{\pi}{6}, \frac{1}{2}\right)$ after correct values seen or implied award A1. Allow e.g. $\theta = \frac{\pi}{6}$, $r = \frac{1}{2}$.<br>**The value of *r* must be seen in (a) – i.e. do not allow recovery in (b).** | | A1 |
| | | | **(4)** |
| **(b)**<br>**Way 1** | Note that the $\frac{1}{2}$ in $\frac{1}{2}\int r^2\,\mathrm{d}\theta$ is not required for the first 4 marks | | |
| | $\int (1-\sin\theta)^2\,\mathrm{d}\theta = \int (1-2\sin\theta+\sin^2\theta)\mathrm{d}\theta$<br>$= \int \left(1-2\sin\theta+\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)\mathrm{d}\theta$ | Attempts $\left(\frac{1}{2}\right)\int r^2\,\mathrm{d}\theta$ and applies $\sin^2\theta = \pm\frac{1}{2} \pm \frac{1}{2}\cos 2\theta$ | M1 |
| | $\int (1-\sin\theta)^2\,\mathrm{d}\theta = \frac{3}{2}\theta + 2\cos\theta - \frac{1}{4}\sin 2\theta (+c)$<br>Correct integration. Condone mixed variables e.g.<br>$\int (1-\sin\theta)^2\,\mathrm{d}\theta = \frac{3}{2}x + 2\cos\theta - \frac{1}{4}\sin 2\theta (+c)$ | | A1 |
| | $\left(\frac{1}{2}\right)\left[\frac{3}{2}\theta + 2\cos\theta - \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{6}} = \left(\frac{1}{2}\right)\left[\left(\frac{\pi}{4}+\sqrt{3}-\frac{\sqrt{3}}{8}\right)-(2)\right]\left(= \frac{\pi}{8}+\frac{7\sqrt{3}}{16}-1\right)$<br>Applies the limits of 0 and their $\frac{\pi}{6}$ to their integration. The $\frac{1}{2}$ is not required.<br>For the integration look for at least $\pm\int \sin\theta\,\mathrm{d}\theta \rightarrow \pm\cos\theta$ | | M1 |
| | Triangle: $\frac{1}{2}\times\frac{1}{2}\sin\frac{\pi}{6}\times\frac{1}{2}\cos\frac{\pi}{6}\left(=\frac{\sqrt{3}}{32}\right)$<br>Uses a correct strategy for the area of the triangle | | M1 |
| | Area of $R = \frac{\pi}{8}+\frac{7\sqrt{3}}{16}-1+\frac{\sqrt{3}}{32}$ | Fully correct method for the required area. **Depends on all previous method marks.** | dM1 |

| | | | |
|---|---|---|---|
| | $\frac{1}{32}\left(4\pi+15\sqrt{3}-32\right)$ | Cao | A1 |
| | | | **(6)** |
| | | | **Total 10** |

| | | | |
|---|---|---|---|
| | Note that the $\frac{1}{2}$ in $\frac{1}{2}\int r^2\,\mathrm{d}\theta$ is not required for the first 3 marks | | |
| **(b) Way 2** | $\int(1-\sin\theta)^2\,\mathrm{d}\theta=\int\left(1-2\sin\theta+\sin^2\theta\right)\mathrm{d}\theta$<br>$=\int\left(1-2\sin\theta+\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)\mathrm{d}\theta$ | Attempts $\left(\frac{1}{2}\right)\int r^2\,\mathrm{d}\theta$ and applies $\sin^2\theta=\pm\frac{1}{2}\pm\frac{1}{2}\cos 2\theta$. | M1 |
| | $\int(1-\sin\theta)^2\,\mathrm{d}\theta=\frac{3}{2}\theta+2\cos\theta-\frac{1}{4}\sin 2\theta(+c)$<br>Correct integration. Condone mixed variables e.g.<br>$\int(1-\sin\theta)^2\,\mathrm{d}\theta=\frac{3}{2}x+2\cos\theta-\frac{1}{4}\sin 2\theta(+c)$ | | A1 |
| | $\left(\frac{1}{2}\right)\left[\frac{3}{2}\theta+2\cos\theta-\frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{2}}=\left(\frac{1}{2}\right)\left[\left(\frac{3\pi}{4}+0-0\right)-(2)\right]\left(=\frac{3\pi}{8}-1\right)$<br>Evidence of use of **both** limits 0 and $\frac{\pi}{2}$ to their integration. The $\frac{1}{2}$ is not required.<br>For the integration look for at least $\pm\int\sin\theta\,\mathrm{d}\theta\rightarrow\pm\cos\theta$ | | M1 |
| | Triangle – "Segment":<br>$\frac{1}{2}\times\frac{1}{2}\sin\frac{\pi}{6}\times\frac{1}{2}\cos\frac{\pi}{6}-\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}(1-\sin\theta)^2\,\mathrm{d}\theta$<br>$\frac{\sqrt{3}}{32}-\frac{1}{2}\left[\frac{3}{2}\theta+2\cos\theta-\frac{1}{4}\sin 2\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(=\frac{15\sqrt{3}}{32}-\frac{\pi}{4}\right)$<br>Uses a fully correct strategy for the area above the curve between *O* and *P*.<br>Requires a correct method for the triangle as in Way 1 and a correct method for the "segment" using **both** their $\frac{\pi}{6}$ and $\frac{\pi}{2}$. | | M1 |
| | Area of $R=\frac{3\pi}{8}-1+\frac{15\sqrt{3}}{32}-\frac{\pi}{4}$ | Fully correct method for the required area. Depends on all previous method marks. | dM1 |
| | $\frac{1}{32}\left(4\pi+15\sqrt{3}-32\right)$ | cao | A1 |
| | | | **(6)** |