Question Number	Scheme	Notes	Marks
9(a)(i)	$x=t^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y}=\frac{1}{2}t^{-\frac{1}{2}}\frac{\mathrm{d}t}{\mathrm{d}y} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=...$ or $t=x^2 \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}x}=2x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}x}=...$ Applies the chain rule and proceeds to an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$		M1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=2t^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t}$	Any correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y and t	A1
(a)(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=2t^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t} \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}y}{\mathrm{d}t}t^{-\frac{1}{2}}\frac{\mathrm{d}t}{\mathrm{d}x}+2t^{\frac{1}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\frac{\mathrm{d}t}{\mathrm{d}x}$ **dM1**: Uses the product rule to differentiate an equation of the form $\frac{\mathrm{d}y}{\mathrm{d}x}=kt^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t}$ or equivalent e.g. $\frac{\mathrm{d}y}{\mathrm{d}x}=kx\frac{\mathrm{d}y}{\mathrm{d}t}$ to obtain $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\alpha t^{-\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}x}+...$ or $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=...+\beta t^{\frac{1}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\frac{\mathrm{d}t}{\mathrm{d}x}$ **<u>or equivalent expressions</u>** where … is non-zero **A1**: <u>Any</u> correct expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$		**d**M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}t}t^{-\frac{1}{2}}\frac{\mathrm{d}t}{\mathrm{d}x}+2t^{\frac{1}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}t^{-\frac{1}{2}}\times 2t^{\frac{1}{2}}+2t^{\frac{1}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\frac{\mathrm{d}t}{\mathrm{d}x}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\frac{\mathrm{d}y}{\mathrm{d}t}+4t\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$ Correct expression in terms of y and t		A1
			(5)
(b)	$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\left(6x^2+1\right)\frac{\mathrm{d}y}{\mathrm{d}x}+9x^3y=x^5 \Rightarrow t^{\frac{1}{2}}\left(2\frac{\mathrm{d}y}{\mathrm{d}t}+4t\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\right)-\left(6t+1\right)2t^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t}+9t^{\frac{3}{2}}y=t^{\frac{5}{2}}$ Substitutes their expressions from part (a) and replaces x with $t^{\frac{1}{2}}$		M1
	$2t^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t}+4t^{\frac{3}{2}}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-12t^{\frac{3}{2}}\frac{\mathrm{d}y}{\mathrm{d}t}-2t^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t}+9t^{\frac{3}{2}}y=t^{\frac{5}{2}}$ $\Rightarrow 4\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-12\frac{\mathrm{d}y}{\mathrm{d}t}+9y=t*$ Obtains the given answer with no errors and sufficient working shown – at least one intermediate line after substitution but check working. **Must follow full marks in (a) apart from SC below.**		A1*
			(2)

Special case in (a) and (b) for those who do not have (a) in terms of *y* and *t* only:

$t = x^2 \Rightarrow \dfrac{\mathrm{d}t}{\mathrm{d}x} = 2x \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t}\dfrac{\mathrm{d}t}{\mathrm{d}x} = ...$ **Scores M1.** $... = 2x\dfrac{\mathrm{d}y}{\mathrm{d}t}$ **scores A0 in (a)(i)**

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2x\dfrac{\mathrm{d}y}{\mathrm{d}t} \Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\dfrac{\mathrm{d}y}{\mathrm{d}t} + 2x\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}\dfrac{\mathrm{d}t}{\mathrm{d}x} = 2\dfrac{\mathrm{d}y}{\mathrm{d}t} + 4x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}$ **Scores dM1A1A0 in (a)(ii)**

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \left(6x^2+1\right)\frac{\mathrm{d}y}{\mathrm{d}x} + 9x^3y = x^5 \Rightarrow t^{\frac{1}{2}}\left(2\frac{\mathrm{d}y}{\mathrm{d}t} + 4x^2\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\right) - \left(6t+1\right)2x\frac{\mathrm{d}y}{\mathrm{d}t} + 9t^{\frac{3}{2}}y = t^{\frac{5}{2}}$$

$$\Rightarrow t^{\frac{1}{2}}\left(2\frac{\mathrm{d}y}{\mathrm{d}t} + 4t\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\right) - \left(6t+1\right)2t^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}t} + 9t^{\frac{3}{2}}y = t^{\frac{5}{2}} \Rightarrow 4\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 12\frac{\mathrm{d}y}{\mathrm{d}t} + 9y = t^* \text{ **Scores M1A1 in (b)**}$$

Mark (c) and (d) together

(c)	$4m^2 - 12m + 9 = 0 \Rightarrow m = \dfrac{3}{2}$	Attempts to solve $4m^2 - 12m + 9 = 0$ Apply general guidance for solving a 3TQ if necessary.	M1
	$\left(y =\right)\mathrm{e}^{\frac{3}{2}t}\left(At + B\right)$	Correct CF. No need for "$y =$…" Condone $\left(y =\right)\mathrm{e}^{\frac{3}{2}x}\left(Ax + B\right)$ here but must be in terms of t in the GS. Allow equivalents for the $\dfrac{3}{2}$.	A1
	$\left(y =\right)at + b \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}t} = a \Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}t^2} = 0$ $\Rightarrow -12a + 9\left(at + b\right) = t$ Starts with the **correct** PI form and differentiates to obtain $\dfrac{\mathrm{d}y}{\mathrm{d}t} = a$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2} = 0$ and substitutes. NB starting with a PI of $y = at$ is M0		M1
	$9a = 1 \Rightarrow a = ...$ $9b - 12a = 0 \Rightarrow b = ...$	Complete method to find a and b by comparing coefficients. **Depends on the previous method mark.**	dM1
	$y = \mathrm{e}^{\frac{3}{2}t}\left(At + B\right) + \dfrac{1}{9}t + \dfrac{4}{27}$	Correct GS including "$y =$…" and must be in terms of t (no x's). Allow equivalent exact fractions for the constants.	A1
			(5)
(d)	$y = \mathrm{e}^{\frac{3}{2}x^2}\left(Ax^2 + B\right) + \dfrac{1}{9}x^2 + \dfrac{4}{27}$ Correct equation including "$y =$…" (follow through their answer to (c)). Allow equivalent exact fractions for the constants. For the ft, the answer to (c) must be in terms of t and the answer to (d) should be the same as (c) with t replaced with x^2. If there is no final answer to (c) you can award B1ft if the equation is correct in terms of x if it follows the previous work.		B1ft
			(1)
			Total 13

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