9. (a) Given that $x = t^{\frac{1}{2}}$, determine, in terms of y and t,

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(ii) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$

(5)

(b) Hence show that the transformation $x = t^{\frac{1}{2}}$, where $t > 0$, transforms the differential equation

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \left(6x^2 + 1\right)\frac{\mathrm{d}y}{\mathrm{d}x} + 9x^3y = x^5 \qquad \text{(I)}$$

into the differential equation

$$4\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 12\frac{\mathrm{d}y}{\mathrm{d}t} + 9y = t \qquad \text{(II)}$$

(2)

(c) Solve differential equation (II) to determine a general solution for y in terms of t.

(5)

(d) Hence determine the general solution of differential equation (I).

(1)

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Question 9 continued

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Question 9 continued

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(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

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