2. The complex number z_1 is defined as

$$z_1 = \frac{\left(\cos\frac{5\pi}{12} + \mathrm{i}\sin\frac{5\pi}{12}\right)^4}{\left(\cos\frac{\pi}{3} - \mathrm{i}\sin\frac{\pi}{3}\right)^3}$$

(a) Without using your calculator show that

$$z_1 = \cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}$$

(4)

(b) Shade, on a single Argand diagram, the region R defined by

$$|z - z_1| \leqslant 1 \quad \textbf{and} \quad 0 \leqslant \arg(z - z_1) \leqslant \frac{3\pi}{4}$$

(4)

Given that the complex number z lies in R

(c) determine the smallest possible positive value of $\arg z$

(2)

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 10 marks)