Question Number	Scheme	Marks
5(a)	$w=\dfrac{z+1}{z-3}\Rightarrow wz-3w=z+1\Rightarrow wz-z=3w+1\Rightarrow z=\dfrac{3w+1}{w-1}$	M1 A1
	$(z=)\dfrac{(3u+1)+3\mathrm{i}v}{(u-1)+\mathrm{i}v}\times\dfrac{(u-1)-\mathrm{i}v}{(u-1)-\mathrm{i}v}=\ldots$	**d**M1
	$\Rightarrow x=\dfrac{3u^2-2u+3v^2-1}{(u-1)^2+v^2}\quad y=\dfrac{-4v}{(u-1)^2+v^2}$ $\left(y=4x\Rightarrow\right)\quad -4v=12u^2-8u+12v^2-4$	**dd**M1
	$\Rightarrow 3u^2+3v^2-2u+v-1=0*$	A1cso*
		(5)
(b)	$u^2-\dfrac{2}{3}u+v^2+\dfrac{1}{3}v-\dfrac{1}{3}=0\Rightarrow\left(u-\dfrac{1}{3}\right)^2-\dfrac{1}{9}+\left(v+\dfrac{1}{6}\right)^2-\dfrac{1}{36}-\dfrac{1}{3}=0$ $\Rightarrow\left(u-\dfrac{1}{3}\right)^2+\left(v+\dfrac{1}{6}\right)^2=\dfrac{17}{36}$ $\Rightarrow$ centre: $\left(\dfrac{1}{3},-\dfrac{1}{6}\right)$ $\left[\text{allow } x=\dfrac{1}{3}, y=-\dfrac{1}{6}\right]$ radius: $\sqrt{\dfrac{17}{36}}$ or $\dfrac{\sqrt{17}}{6}$	B1 B1
		(2)
		Total 7

Notes

(a)

M1: Completes an attempt to make *z* the subject.

A1: Correct expression.

dM1: **Dependent on previous M mark.** Replaces *w* with *u* + i*v* and indicates an appropriate attempt to rationalise the denominator. Accept if *x* + i*y* is used instead for this mark.

ddM1: **Dependent on both previous M marks.** Equates real and imaginary parts to obtain expressions for *x* and *y* in *u* and/or *v* only and uses *y* = 4*x* to form an equation in *u* and *v* only. Condone intermediate incorrect statements that are recovered for this mark, e.g. condone, missing or incorrect (real) denominator or slips with initial inclusion of i in the *y* if recovered. E.g. any of

$x=3u^2-2u+3v^2-1,\quad x=\dfrac{3u^2-2u+3v^2-1}{(u-1)^2-v^2},\quad y=\dfrac{-4v\mathrm{i}}{(u-1)^2+v^2}$ or similar may be recovered for the M's

A1cso*:obtains the equation (must be equal to 0) or states *k* = –1 following correct work. Do not allow recovery from incorrect statements as shown above for the final mark - all may lead to the correct answer but are A0. But if there are no incorrect statements e.g. cancelling of denominators may be implied for the A. Watch out for an incorrect $z=\dfrac{3w+1}{1-w}$ which also leads to a correct expression but loses both A marks.

Note: Alternatively methods are possible. Two are shown on the next page, but are not exhaustive.

(b)

Allow both marks from cases where *k* = −1 is guessed or achieved from fortuitous work.

B1: Correct centre **or** radius. Accept unsimplified fractions for this mark.

B1: Correct centre **and** radius - fractions must be simplified.

5(a) Alt	$z = x + \mathrm{i}y,\ y = 4x \Rightarrow w = \dfrac{z+1}{z-3} = \dfrac{x+4x\mathrm{i}+1}{x+4x\mathrm{i}-3} = \dfrac{(x+1)+4\mathrm{i}x}{(x-3)+4\mathrm{i}x} \times \dfrac{(x-3)-4\mathrm{i}x}{(x-3)-4\mathrm{i}x} = \ldots$	M1
	$(u+\mathrm{i}v=)\dfrac{\left(17x^2-2x-3\right)-16\mathrm{i}x}{(x-3)^2+16x^2}$ oe	**A1**
	$\Rightarrow u = \dfrac{17x^2-2x-3}{(x-3)^2+16x^2} \quad v = \dfrac{-16x}{(x-3)^2+16x^2}$ $\Rightarrow 3u^2+3v^2-2u+v=\ldots$	**d**M1
	$\Rightarrow \ldots \Rightarrow 3u^2+3v^2-2u+v-1=0^*$ or states $k=-1$	**dd**M1A1cso*
		(5)
5(a) Alt II	$z = x + \mathrm{i}y,\ y = 4x \Rightarrow w = \dfrac{z+1}{z-3} = \dfrac{x+4x\mathrm{i}+1}{x+4x\mathrm{i}-3}$	
	$\Rightarrow (u+\mathrm{i}v)\left((x-3)+4\mathrm{i}x\right) = (x+1)+4\mathrm{i}x \Rightarrow \ldots + \ldots\mathrm{i} = \ldots$	M1
	$u(x-3)-4vx+\mathrm{i}\left(v(x-3)+4ux\right) = (x+1)+4\mathrm{i}x$	A1
	$\Rightarrow u(x-3)-4vx=(x+1) \quad v(x-3)+4ux=4x$ $\Rightarrow x = \dfrac{3u+1}{u-4v-1} \quad x = \dfrac{3v}{v+4u-4} \Rightarrow \dfrac{3u+1}{u-4v-1} = \dfrac{3v}{v+4u-4}$	**d**M1
	$(3u+1)(v+4u-4) = 3v(u-4v-1) \Rightarrow 12u^2+12v^2-8u+4v-4=0$ $\Rightarrow 3u^2+3v^2-2u+v-1=0^*$	**dd**M1A1cso*
		(5)

Notes

(a) Alt I

M1: Uses $y = 4x$ and $z = x + \mathrm{i}y$ in the expression for w and multiplies through by complex conjugate of denominator

A1: Correct expression in terms of x (oe)

dM1: Replaces w with $u + \mathrm{i}v$ and equates real and imaginary parts and substitutes into the LHS of given equation. **Dependent on previous M mark.**

ddM1: Proceeds simplify the expression putting over common denominator.They are unlikely to make much progress at this stage in this method. **Dependent on both previous M marks.**

A1cso*: Correctly shows the expression equates to 0 and gives minimal conclusion.

(a) Alt II

M1: Uses $y = 4x$ and $z = x + \mathrm{i}y$ in the expression for w and equates to $u + \mathrm{i}v$, cross multiples and expands.

A1: Correct expression with the i^2 terms simplified.

dM1: Equates real and imaginary parts and proceeds to eliminate x solving simultaneously. **Dependent on previous M mark.**

ddM1: Proceeds to simplify by cross multiplying and expanding to at least eliminate uv terms. **Dependent on both previous M marks.**

A1cso*: Correctly shows the expression required holds. As main scheme.