5. The transformation T from the z-plane, where $z = x + iy$, to the w-plane, where $w = u + iv$ is given by

$$w = \frac{z+1}{z-3} \qquad z \neq 3$$

The straight line in the z-plane with equation $y = 4x$ is mapped by T onto the circle C in the w-plane.

(a) Show that C has equation

$$3u^2 + 3v^2 - 2u + v + k = 0$$

where k is a constant to be determined.

(5)

(b) Hence determine

(i) the coordinates of the centre of C

(ii) the radius of C

(2)

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 7 marks)