Question Number	Scheme	Marks
6(a)	$y=\sec x \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\sec x\tan x$ $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=(\sec x\tan x)\tan x+\sec x\left(\sec^2 x\right)\ \left(=\sec x\tan^2 x+\sec^3 x\right)$	M1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\sec x\left(\sec^2 x-1\right)+\sec^3 x=2\sec^3 x-\sec x$ $\Rightarrow \dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=6\sec^2 x\left(\sec x\tan x\right)-\sec x\tan x$ **or** $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=2\sec x\tan x\sec^2 x+\tan^2 x\sec x\tan x+3\sec^2 x\sec x\tan x$	**d**M1
	$\left(\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=5\sec^3 x\tan x+\sec x\tan x\left(\sec^2 x-1\right)=6\sec^3 x\tan x-\sec x\tan x\right)$ $\Rightarrow \dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=\sec x\tan x\left(6\sec^2 x-1\right)$	**dd**M1 A1
		(4)
(b)	$\sec\left(\dfrac{\pi}{3}\right)=2,\ \tan\left(\dfrac{\pi}{3}\right)=\sqrt{3}\Rightarrow \mathrm{f}\left(\dfrac{\pi}{3}\right)=2,\ \mathrm{f}'\left(\dfrac{\pi}{3}\right)=2\sqrt{3},\ \mathrm{f}''\left(\dfrac{\pi}{3}\right)=14,\ \mathrm{f}'''\left(\dfrac{\pi}{3}\right)=46\sqrt{3}$	M1
	$\mathrm{f}(x)=\mathrm{f}(a)+(x-a)\mathrm{f}'(a)+\dfrac{(x-a)^2}{2!}\mathrm{f}''(a)+\dfrac{(x-a)^3}{3!}\mathrm{f}'''(a)+\ldots$ $\Rightarrow 2+2\sqrt{3}\left(x-\dfrac{\pi}{3}\right)+7\left(x-\dfrac{\pi}{3}\right)^2+\dfrac{23\sqrt{3}}{3}\left(x-\dfrac{\pi}{3}\right)^3$	**d**M1 A1
		(3)
(c)	$\left(\sec\dfrac{7\pi}{24}\approx\right)=2+2\sqrt{3}\left(-\dfrac{\pi}{24}\right)+7\left(-\dfrac{\pi}{24}\right)^2+\dfrac{23\sqrt{3}}{3}\left(-\dfrac{\pi}{24}\right)^3=\ldots$	M1
	$\sec\dfrac{7\pi}{24}\approx 1.636709263\ldots\approx 1.637$	A1
		(2)
		Total 9

Notes

(a)

M1: Differentiates twice and obtains expression for second derivative allowing for sign errors only

dM1: **Dependent on previous M mark.** Differentiates again and obtains expression for third derivative allowing for sign errors only (which could come from incorrect signs in trig identities)

ddM1: **Dependent on both previous M marks.** Obtains an answer of the correct form or values for *p* and *q*. Allow slips from their third derivative but any trig identities used can only have errors in sign.

A1: Fully correct expression or values for *p* and *q*

Note: If they decide not to work in sec *x* and tan *x* the main scheme still applies – see Alt next page.

Allow "meet in the middle" approaches that determine the values of *p* and *q*

(b)

M1: Uses sec *x* and their three derivatives to attempt to find values for $\mathrm{f}\left(\dfrac{\pi}{3}\right),\ \mathrm{f}'\left(\dfrac{\pi}{3}\right),\ \mathrm{f}''\left(\dfrac{\pi}{3}\right)$ and $\mathrm{f}'''\left(\dfrac{\pi}{3}\right)$ evidenced by values for each with at least 2 correct if no method seen. May be recovered by substitution of values into formula if not all listed.

dM1: **Dependent on previous M mark.** Uses a correct Taylor series with all four of their values. Series must be correct for their values but allow slips if a correct formula is quoted.

A1: Correct series with coefficients in a simplest form e.g., allow $\sqrt{12}$ for $2\sqrt{3}$, $\frac{23}{\sqrt{3}}$ for $\frac{23}{3}\sqrt{3}$ Condone absence of sec x =... or y =… Must have had a correct third derivative in (a), though need not have been simplified to the form required (and allow if there was a correct third derivative which was incorrectly simplified if the correct answer is found).

(c)

M1: Shows evidence of substitution of $\frac{7\pi}{24}$ into their series of the right form (powers of $\left(x-\frac{\pi}{3}\right)$). If only a value is given score M0 unless it is the correct 4 s.f. value for their series (1.636709263... allowing awrt 1.637 if (b) is correct). You may need to check the answer.

A1: 1.637 only (not awrt) from a correct series.

Note that $\sec\frac{7\pi}{24}$ = 1.642679632...

6(a) Alt	$y=\sec x \Rightarrow y=(\cos x)^{-1} \Rightarrow \frac{dy}{dx}=-(\cos x)^{-2}(-\sin x)=\frac{\sin x}{\cos^2 x}$ $\frac{d^2y}{dx^2}=\frac{\cos^2 x\cos x-2\sin x\cos x(-\sin x)}{\cos^4 x}$	M1
	$\frac{d^2y}{dx^2}=\frac{\cos^3 x+2\cos x(1-\cos^2 x)}{\cos^4 x}=\frac{2\cos x-\cos^3 x}{\cos^4 x}=2(\cos x)^{-3}-(\cos x)^{-1}$ $\Rightarrow \frac{d^3y}{dx^3}=-6(\cos x)^{-4}(-\sin x)+(\cos x)^{-2}(-\sin x)$	**d**M1
	$\frac{d^3y}{dx^3}=\frac{6\sin x}{\cos^4 x}-\frac{\sin x}{\cos^2 x}=\frac{\sin x}{\cos^2 x}\left(\frac{6}{\cos^2 x}-1\right)=\sec x\tan x(6\sec^2 x-1)$	**dd**M1 A1
		(4)

Notes

Mark as per main scheme allowing sign slips only - forms correct at each stage.