Question Number	Scheme	Marks
7(a)	$z = y^{-2} \Rightarrow$ $\quad \dfrac{\mathrm{d}z}{\mathrm{d}y} = -2y^{-3}$ or $1 = -2y^{-3}\dfrac{\mathrm{d}y}{\mathrm{d}z}$ $\qquad y^2 = z^{-1} \Rightarrow$ $\quad 2y\dfrac{\mathrm{d}y}{\mathrm{d}z} = -z^{-2}$ $\qquad y = z^{-\frac{1}{2}} \Rightarrow$ $\quad \dfrac{\mathrm{d}y}{\mathrm{d}z} = -\dfrac{1}{2}z^{-\frac{3}{2}}$	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}z}{\mathrm{d}x}\cdot\dfrac{\mathrm{d}y}{\mathrm{d}z}$ (oe) $\Rightarrow$ e.g., $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{2}y^3\dfrac{\mathrm{d}z}{\mathrm{d}x}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{2}z^{-\frac{3}{2}}\dfrac{\mathrm{d}z}{\mathrm{d}x}$	M1 A1
	$x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y + 4x^2y^3\ln x = 0 \Rightarrow$ e.g., $-\dfrac{1}{2}xy^3\dfrac{\mathrm{d}z}{\mathrm{d}x} + y + 4x^2y^3\ln x = 0$ $\qquad -\dfrac{1}{2}xz^{-\frac{3}{2}}\dfrac{\mathrm{d}z}{\mathrm{d}x} + z^{-\frac{1}{2}} + 4x^2z^{-\frac{3}{2}}\ln x = 0$	**d**M1
	$\dfrac{\mathrm{d}z}{\mathrm{d}x} - \dfrac{2}{xy^2} - 8x\ln x = 0$ $\qquad \dfrac{\mathrm{d}z}{\mathrm{d}x} - \dfrac{2z}{x} - 8x\ln x = 0$ $\Rightarrow \dfrac{\mathrm{d}z}{\mathrm{d}x} - \dfrac{2z}{x} = 8x\ln x$ *	A1*
		(5)
(b)	$\left(\mathrm{IF} = \right)\ \mathrm{e}^{\int -\frac{2}{x}(\mathrm{d}x)}$	M1
	$= \mathrm{e}^{-2\ln x}$ $\quad \left(= x^{-2}\right)$	A1
	$x^{-2}z = \int x^{-2}\left(8x\ln x\right)[\mathrm{d}x]$	M1
	E.g. Parts: $\int x^{-1}\ln x\,\mathrm{d}x$: $\quad \left(u = \ln x,\ u' = x^{-1}, v' = x^{-1}, v = \ln x\right)$ $\Rightarrow I = \left(\ln x\right)^2 - kI \Rightarrow I = p\left(\ln x\right)^2$ Or substitution: $t = \ln x$, $\dfrac{\mathrm{d}t}{\mathrm{d}x} = x^{-1} \Rightarrow I = k\int x^{-1}\ln x\cdot x\,\mathrm{d}t = k\int t\,\mathrm{d}t = pt^2$	M1
	$\int x^{-1}\ln x\,\mathrm{d}x = \dfrac{1}{2}\left(\ln x\right)^2\ [+c]$	A1
	$x^{-2}z = 4\left(\ln x\right)^2 + k \Rightarrow z = 4x^2\left(\ln x\right)^2 + kx^2$ $\Rightarrow y^2 = \dfrac{1}{4x^2\left(\ln x\right)^2 + kx^2}$ oe	A1
		(6)
		Total 11

Notes

(a)

B1: Any correct equation following differentiation of the given substitution. Could be implied.

M1: Uses a correct chain rule to obtain an equation linking $\dfrac{\mathrm{d}y}{\mathrm{d}x}\left(\text{or }\dfrac{\mathrm{d}x}{\mathrm{d}y}\right)$ **and** $\dfrac{\mathrm{d}z}{\mathrm{d}x}\left(\text{or }\dfrac{\mathrm{d}x}{\mathrm{d}z}\right)$

A1: Any correct equation

Note that the first three marks could be scored by a first step of, e.g.:

$$\frac{\mathrm{d}z}{\mathrm{d}x} = -2y^{-3}\frac{\mathrm{d}y}{\mathrm{d}x} \quad \text{or} \quad 2y\frac{\mathrm{d}y}{\mathrm{d}x} = -z^{-2}\frac{\mathrm{d}z}{\mathrm{d}x} \quad \text{or} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}z^{-\frac{3}{2}}\frac{\mathrm{d}z}{\mathrm{d}x}$$

dM1: **Dependent on previous M mark.** Substitutes their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ into differential equation (I). Need not replace fully for this mark.

A1*: Fully correct proof with no errors. There must be an intermediate line of working between the line where they substitute into DE(I) and the given answer.

Note: Use Review for any incorrect but potentially creditworthy attempts that "meet in the middle" or use (II) $\Rightarrow$ (I)

(b)

M1: Correct form for integrating factor. Allow $e^{\int \frac{k}{x}}$ and missing dx

A1: Correct IF in any form

M1: Obtains $\text{IF} \cdot z = \int \text{IF} \cdot (8x \ln x)[\text{d}x]$ Correct for their IF and z not y

M1: Must have achieved an integral of form $k\int x^{-1}(\ln x)[\text{d}x]$. Correct form following integration of $x^{-1}\ln x$ (constant not required):

$$\int x^{-1} \ln x \, \text{d}x \Rightarrow p(\ln x)^2 \ [+c]$$

The work in the scheme above is sufficient but you do not need to scrutinise the details, accept work that leads to the correct form.. Condone " $\ln x^2$ " for this mark

A1: Correct integration (constant not required) $\ln x^2$ is A0 unless recovered – must see $(\ln x)^2$ but allow $\ln^2 x$

A1: Any correct equation in $y^2 = ...$ form e.g., $y^2 = \left[x^2\left(4\ln^2 x + k\right)\right]^{-1}$ Constant may appear as a multiple, e.g., 8c