Question Number	Scheme	Marks
8	$r = 6(1 + \cos\theta) \qquad 0 \leqslant \theta \leqslant \pi$	
(a)	$\theta = 0,\ r = 6(1 + \cos 0) \Rightarrow 12$ or $(12, 0)$	B1
		(1)
(b)	$\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta)$: $= 6\sin\theta(1+\cos\theta) \Rightarrow$ $6\sin\theta(-\sin\theta)+6\cos\theta(1+\cos\theta)$ $= 6\sin\theta + 6\sin\theta\cos\theta \Rightarrow$ $6\cos\theta + 6(\cos^2\theta - \sin^2\theta)$ $= 6\sin\theta + 3\sin 2\theta \Rightarrow$ $6\cos\theta + 6\cos 2\theta$	M1
	$\Rightarrow 2\cos^2\theta + \cos\theta - 1 = 0$ $\left[(2\cos\theta - 1)(\cos\theta + 1) = 0 \Rightarrow\right] \quad \cos\theta = \frac{1}{2} \quad [\text{or } -1]$	**d**M1
	$\theta = \frac{\pi}{3} \qquad r = 6\left(1 + \cos\frac{\pi}{3}\right) = 9 \qquad \text{or} \quad \left(9, \frac{\pi}{3}\right)$	A1 A1
		(4)
8(c)	$\left[\tfrac{1}{2}\right]\int r^2\,\mathrm{d}\theta = \left[\tfrac{1}{2}\right]\int 36(1+\cos\theta)^2\ [\mathrm{d}\theta]$	M1
	$\int r^2\,\mathrm{d}\theta = 36\int(1 + 2\cos\theta + \cos^2\theta)[\mathrm{d}\theta] = 36\int\left(\tfrac{3}{2} + 2\cos\theta + \tfrac{1}{2}\cos 2\theta\right)[\mathrm{d}\theta]$	M1
	$\int(a + b\cos\theta + c\cos 2\theta)[\mathrm{d}\theta] = a\theta \pm b\sin\theta \pm \tfrac{c}{2}\sin 2\theta$	M1
	$18\left[\tfrac{3}{2}\theta + 2\sin\theta + \tfrac{1}{4}\sin 2\theta\right]_{\cdots}^{\cdots}$	A1
	$18\left[\tfrac{3}{2}\theta + 2\sin\theta + \tfrac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{3}} = 18\left(\frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8}\right)\ [-0] \qquad \left[= 9\pi + \frac{81}{4}\sqrt{3}\right]$	**d**M1
	E.g. $OB = 6\left(1 + \cos\frac{'\pi'}{3}\right) \Rightarrow BQ = 6\left(1 + \cos\frac{'\pi'}{3}\right)\sin\frac{'\pi'}{3} \quad \left[= \frac{9\sqrt{3}}{2}\right]$	M1
	$BP = 12 - 9\cos\frac{\pi}{3} = 12 - \frac{9}{2} = \frac{15}{2}$ $\Rightarrow \text{area } OBPA = \frac{1}{2}\left(12 + \frac{15}{2}\right)\left(\frac{9\sqrt{3}}{2}\right)$ or $\frac{1}{2}\times\frac{9}{2}\times\frac{9\sqrt{3}}{2} + \frac{9\sqrt{3}}{2}\times\frac{15}{2} \quad \left[= \frac{351}{8}\sqrt{3}\right]$	A1
	area of region $R = \frac{351}{8}\sqrt{3} - \frac{81}{4}\sqrt{3} - 9\pi = \frac{189}{8}\sqrt{3} - 9\pi$	A1
		(8)
		Total 13

Notes

(a)

B0: Correct values for θ and r or correct coordinates. Condone (0, 12).

(b)

M1: Differentiates $r\sin\theta$. Allow sign errors only. The "6" may be missing

dM1:**Dependent on previous M mark.** Uses correct identity/identities to reach a 3TQ in $\cos\theta$ and solves.

Apply usual rules or by calculator must obtain at least one real consistent solution where $-1 \leqslant \cos\theta \leqslant 1$.

Accept $\theta = \frac{\pi}{3}$ following a correct derivative set equal to zero to imply the M if no incorrect working shown (by calculator or by inspection).

A1: Either r or θ correct.

A1: Both coordinates correct. Only accept $\frac{\pi}{3}$ and 9. Withhold last mark if additional answers offered and not rejected, but isw after correct answers seen if only a miscopy is made.

(c)

M1: Attempts $\int r^2 \, \mathrm{d}\theta$ which may include the $\frac{1}{2}$ for the area formula. The multiple 36 may be missing or wrong. Condone poor squaring - allow this mark if there are only two terms.

M1: Uses $\cos^2\theta = \pm\frac{1}{2}\cos 2\theta \pm \frac{1}{2}$ and obtains an integrand of the form $a + b\cos\theta + c\cos 2\theta$ (constants may be uncollected)

M1: Integrates and obtains a form $a\theta \pm b\sin\theta \pm \frac{c}{2}\sin 2\theta$ (sign errors on trig terms only). May be two terms in θ

A1: Fully correct expression for the area of the sector after integration. Ignore limits but $\frac{1}{2}$ must have been used or appear later.

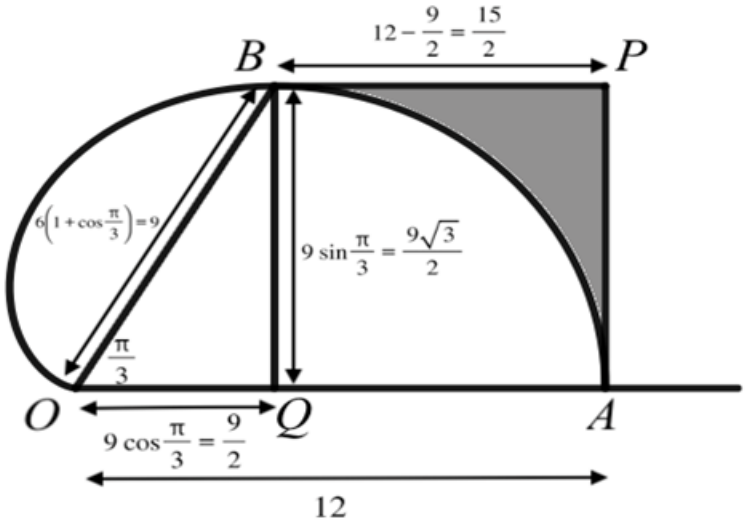
dM1: **Dependent on previous M mark.** Evidence of substitution of "correct" limits into the integral, so their $\frac{\pi}{3}\left(\text{provided } 0 < \theta < \frac{\pi}{2}\right)$ and the lower limit must be 0 and lead to zero but this can be implied by omission.

M1: Correct method for the perpendicular distance between l_1 and the initial line with their r and θ $\left(\text{provided } 0 < \theta < \frac{\pi}{2}\right)$ for point *B.* This may only be seen embedded in their attempt at an area, so may be implied if not explicit. E.g. look for "their r sin their $\frac{\pi}{3}$" or may find r again from the equation of curve.

A1: Correct expression for area of trapezium *OBPA* - may be given as as a sum of separate areas, e.g. triangle *OBQ* + rectangle *QBPA.* Other formulations are possible.

A1: Correct answer in the correct form or $p = \frac{189}{8}$ or exact equivalent, $q = -9$

Useful diagram:



	TOTAL FOR PAPER: 75 MARKS

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