Question Number	Scheme	Notes	Marks
2(a)	**(i)** $z = 6 - 6\sqrt{3}\text{i} \Rightarrow \lvert z \rvert = \sqrt{6^2 + \left(6\sqrt{3}\right)^2} = 12$	+12 only. Accept if just stated	**B1**
	(ii) e.g., $\arg z = -\arctan\dfrac{6\sqrt{3}}{6}$ Attempts an expression for a relevant angle. Look for $\pm\arctan\left(\pm\dfrac{6\sqrt{3}}{6}\right)$ or e.g., $\pm\tan^{-1}\left(\pm\dfrac{1}{\sqrt{3}}\right)$ If arctan is not seen allow e.g., $\tan\alpha = \dfrac{6\sqrt{3}}{6} \Rightarrow \alpha = \dfrac{\pi}{3}$ with α correct for their $\tan\alpha$ If using sin or cos the hypotenuse must be their 12		**M1**
	$\arg z$ or arg or argument $\left(\text{of } z\right) = -\dfrac{\pi}{3}$ * A correct proof with no incorrect work/statements. LHS required. **Allow** "$\theta =$" **if consistent**, e.g., $\theta = -\frac{\pi}{3}$ cannot follow "$\tan\theta = +\sqrt{3}$"		**A1***
(ii) Way 2	$z = 12\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\text{i}\right) = 12\left(\cos\left(-\frac{\pi}{3}\right) + \text{i}\sin\left(-\frac{\pi}{3}\right)\right)$ or $12\text{e}^{-\frac{\pi}{3}\text{i}}$ or $\cos\theta = \frac{1}{2}$ or $\sin\theta = -\frac{\sqrt{3}}{2}$ [M1] $\Rightarrow \arg z = -\frac{\pi}{3}$ [A1*] M1: Factorises out 12 and writes in trig or exp form or identifies $\cos\theta = \frac{1}{2}$ and $\sin\theta = -\frac{\sqrt{3}}{2}$ A1: Acceptable statement with all work correct		
(ii) Way 3	$z = 12\left(\cos\left(-\frac{\pi}{3}\right) + \text{i}\sin\left(-\frac{\pi}{3}\right)\right)$ or $12\text{e}^{-\frac{\pi}{3}\text{i}}$ or $12\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\text{i}\right) = 6 - 6\sqrt{3}\text{i}$ [M1] $\Rightarrow \arg z = -\frac{\pi}{3}$ [A1*] M1: Assumes result, writes correctly for their 12 and attempts *a* + i*b* form A1: Obtains $6 - 6\sqrt{3}\text{i}$ and makes acceptable statement with all work correct		
			(3)
(b)	$z = \text{"12"}\left(\cos\left(-\dfrac{\pi}{3}\right) + \text{i}\sin\left(-\dfrac{\pi}{3}\right)\right)$ or $\text{"12"}e^{-\frac{\pi}{3}\text{i}}$ [no missing "i" unless recovered] Correct trig or exp. form with their 12. Could be implied by their z^4 in trig or exp. form e.g., $(\text{"12"}e^{-\frac{\pi}{3}\text{i}})^4$ Allow equivalent values of θ e.g. $\frac{5\pi}{3}$ and use of e.g., $\sin\left(-\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right)$. Condone poor bracketing. Allow this mark if $+2k\pi, -2k\pi, \pm 2k\pi$ appears with argument		**M1**
	$z^4 = 20736\left(\cos\left(-\dfrac{4\pi}{3}\right) + \text{i}\sin\left(-\dfrac{4\pi}{3}\right)\right)$ or $20736\left(\cos -\dfrac{4\pi}{3} + \text{i}\sin -\dfrac{4\pi}{3}\right)$ or $20736e^{-\frac{4\pi}{3}\text{i}}$ Correct z^4 in any form. 12^4 evaluated and arg. of $-\frac{4\pi}{3}$ (not just $4\times-\frac{\pi}{3}$) or $\frac{2\pi}{3}$ only although may use e.g., $\sin\left(-\frac{4\pi}{3}\right) = -\sin\left(\frac{4\pi}{3}\right)$. No "*k*"s. Condone an "unclosed" bracket. Only accept $-10368 + 10368\sqrt{3}\text{i}$ or $20736\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\text{i}\right)$ **provided evidence of de Moivre.**		**A1**
			(2)

Question Number	Scheme	Notes	Marks
2(c)	$w=z^{\frac{1}{2}}=(\pm)\sqrt{"12"}\left(\cos\left(\frac{-\frac{\pi}{3}}{2}\right)+\mathrm{i}\sin\left(\frac{-\frac{\pi}{3}}{2}\right)\right)$ or e.g., $(\pm)"2\sqrt{3}"e^{-\frac{\pi}{6}\mathrm{i}}$ [no missing "i" unless recovered] Correct use of de Moivre's theorem with $-\frac{\pi}{3}$ and their 12 to attempt one square root. Allow work with argument of $\frac{5\pi}{3}$ for $-\frac{\pi}{3}$ and use of e.g., $\sin\left(-\frac{\pi}{6}\right)=-\sin\left(\frac{\pi}{6}\right)$. Condone poor bracketing. M0 if z^4 used for z. Allow this mark if $+2k\pi,-2k\pi,\pm 2k\pi$ appears with argument		**M1**
	$w=3-\sqrt{3}\mathrm{i},-3+\sqrt{3}\mathrm{i}$ oe A1ft: One correct exact root in $a+\mathrm{i}b$ or $c(a+\mathrm{i}b)$ form (a, b, c may be unsimplified but not numerical trig expressions) **ft their 12 only** i.e. $(\pm)\sqrt{"12"}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)$ A1: Both exact roots (no others) correct in $a+\mathrm{i}b$ form – a and b may be unsimplified (but not numerical trig expressions) e.g. accept $a=(\pm)\sqrt{12}\frac{\sqrt{3}}{2}, (\pm)\frac{\sqrt{36}}{2}$ $b=(\mp)\frac{\sqrt{12}}{2}, (\mp)\frac{2\sqrt{3}}{2}$ Accept $\pm\left(3-\sqrt{3}\mathrm{i}\right)$ but just $\pm 3-\sqrt{3}\mathrm{i}$ is A1 A0. Just $\pm\sqrt{3}\left(\sqrt{3}-\mathrm{i}\right)$ is A1 A0		**A1ft** **A1**
	Note: $w^2=r^2(\cos 2\theta+i\sin 2\theta)=z\Rightarrow r,\ \theta,\ w=\ldots$ is an acceptable approach		**(3)**
Alt	$w^2=z\Rightarrow(a+\mathrm{i}b)^2=a^2-b^2+2ab\mathrm{i}=6-6\sqrt{3}\mathrm{i}\Rightarrow a^2-b^2=6,\ 2ab=-6\sqrt{3}$ $b=-\frac{3\sqrt{3}}{a}\Rightarrow a^2-\frac{27}{a^2}=6\Rightarrow a^4-6a^2-27=\left(a^2-9\right)\left(a^2+3\right)=0\Rightarrow a^2=9,\ a=\pm 3,\ b=\mp\sqrt{3}$ **M1**: From a correct starting point, expands and equates real and imaginary parts to form two equations in a and b and obtains at least one value for both a and b $w=3-\sqrt{3}\mathrm{i},-3+\sqrt{3}\mathrm{i}$ **A1**: One correct exact root in $a+\mathrm{i}b$ or $c(a+\mathrm{i}b)$ form (a, b, c may be unsimplified) **A1**: Both exact roots (no others) correct in $a+\mathrm{i}b$ form – a and b may be unsimplified		
			Total 8