Question Number	Scheme	Notes	Marks
3(a)	$\dfrac{r}{\sqrt{r(r+1)}+\sqrt{r(r-1)}}\times\dfrac{\sqrt{r(r+1)}-\sqrt{r(r-1)}}{\sqrt{r(r+1)}-\sqrt{r(r-1)}}$	A correct multiplier to rationalise the denominator seen or implied by correct work	**M1**
	$=\dfrac{r\left(\sqrt{r(r+1)}-\sqrt{r(r-1)}\right)}{r(r+1)-r(r-1)}=\dfrac{\sqrt{r(r+1)}-\sqrt{r(r-1)}}{2}$ or $A=\dfrac{1}{2}$ Correct expression or correct value for *A*. Condone poor notation if intention clear. There must be (minimal) correct supporting working.		**A1**
	Alternative: $A=\dfrac{r}{\left(\sqrt{r(r+1)}+\sqrt{r(r-1)}\right)\left(\sqrt{r(r+1)}-\sqrt{r(r-1)}\right)}=\dfrac{r}{r\left(r+1\right)-r\left(r-1\right)}$ or $\dfrac{r}{r^2+r-r^2+r}$ or $\dfrac{r}{2r}\Rightarrow A=\dfrac{1}{2}$ M1: Correctly makes *A* the subject A1: Correct completion with one intermediate fraction		
			(2)
(b)	$\sum_{r=1}^{n}\dfrac{r}{\sqrt{r(r+1)}+\sqrt{r(r-1)}}=\text{"}\dfrac{1}{2}\text{"}\left(\begin{array}{c}\sqrt{1\times2}-\sqrt{1\times0}\ \left(=\sqrt{2}\ (-0)\right)\\ +\sqrt{2\times3}-\sqrt{2\times1}\ \left(=\sqrt{6}-\sqrt{2}\right)+\ldots\\ \ldots+\sqrt{\left(n-1\right)(n-1+1)}-\sqrt{\left(n-1\right)(n-1-1)}\ \left(=\sqrt{n\left(n-1\right)}-\sqrt{\left(n-1\right)\left(n-2\right)}\right)\\ +\sqrt{n(n+1)}-\sqrt{n(n-1)}\end{array}\right)$ **M1**: Applies the method of differences for $r=1$ and $r=n$ in the given expression with or without their *A* and obtains one correct row of these 2. **M1**: Applies the method of differences for $r=1$, $r=n$ and **either** $r=2$ or $r=n-1$ in the given expression with/without their *A* and obtains 2 correct rows of these 4. When considering how many rows are correct, if *A* has been clearly applied to any term then assess all rows as if *A* has been applied throughout. Condone missing bracket if their *A* is applied to a row e.g., "$\frac{1}{2}\times\sqrt{6}-\sqrt{2}$" <u>if it is recovered</u> but e.g., $\frac{\sqrt{6}}{2}-\sqrt{2}$ is an incorrect row. Ignore a row for $r=0$. Condone equivalent work with *r* or e.g., *k* used for *n*. **Both marks can be implied by a correct final expression with or without their *A* provided there are at least any two correct rows of differences** i.e., $\text{"}\frac{1}{2}\text{"}\left(\sqrt{n(n+1)}-0\right)$ or $\sqrt{n(n+1)}-0$	**M1** **M1**
	Note: row 3 is $\text{"}\frac{1}{2}\text{"}(\sqrt{12}\,(\text{or }2\sqrt{3})-\sqrt{6})$, row 4 is $\text{"}\frac{1}{2}\text{"}(\sqrt{20}\,(\text{or }2\sqrt{5})-\sqrt{12}(\text{or }2\sqrt{3}))$ If $\frac{1}{2}$ is fully applied the rows are: $\frac{\sqrt{2}}{2},\ \frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2},\ \frac{\sqrt{12}}{2}(\text{or}\sqrt{3})-\frac{\sqrt{6}}{2},\ \frac{\sqrt{20}}{2}(\text{or}\sqrt{5})-\frac{\sqrt{12}}{2}(\text{or}\sqrt{3}),\ldots$ $\ldots,\frac{\sqrt{(n-2)(n-1)}}{2}-\frac{\sqrt{(n-2)(n-3)}}{2},\ \frac{\sqrt{n(n-1)}}{2}-\frac{\sqrt{(n-1)(n-2)}}{2},\ \frac{\sqrt{n(n+1)}}{2}-\frac{\sqrt{n(n-1)}}{2}$		
	$=\dfrac{1}{2}\sqrt{n(n+1)}$ oe e.g., $\dfrac{\sqrt{n^2+n}}{2}$	Correct expression in terms of *n*. No incorrect terms seen in differences work even if cancelled but condone the occasional poor bracket. There should be no "0" so e.g., $\frac{1}{2}\left(\sqrt{n(n+1)}-0\right)$ is A0 Does not require marks in (a)	**A1**
			(3)

Question Number	Scheme	Notes	Marks
3(c)	$\sum r = \frac{1}{2}n\left(n+1\right)$ e.g., sight of $k\times ... = \sqrt{\frac{1}{2}n(n+1)}$	States or uses the correct summation formula for integers	**M1**
	$\frac{k}{2}\sqrt{n(n+1)} = \sqrt{\frac{1}{2}n(n+1)} \Rightarrow \frac{k}{2} = \sqrt{\frac{1}{2}} \Rightarrow k = \sqrt{2}$	$\sqrt{2}$ only (Not $\pm$). $k = \sqrt{2}$ must not come from a clearly incorrect equation.	**A1**
			(2)
			Total 7