Question Number	Scheme	Notes	Marks
6	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+6\frac{\mathrm{d}x}{\mathrm{d}t}+13x=8\mathrm{e}^{-3t}$ $\quad t\ldots 0$		
(a)	$m^2+6m+13=0\Rightarrow m=\frac{-6\pm\sqrt{36-52}}{2}$ $\left\{=-3\pm 2\mathrm{i}\right\}$	Forms correct auxiliary equation and obtains a correct numerical expression for at least one root by formula or uses CTS (apply usual CTS rule below). One correct root if no working	**M1**
	CTS rule : $m^2+6m+13=0\Rightarrow\left(m\pm\frac{6}{2}\right)^2\pm q\pm 13=0,\ q\neq 0\Rightarrow m=...$		
	CF examples: $(x=)\ \mathrm{e}^{-3t}\left(A\cos 2t+B\sin 2t\right)$ or $(x=)\ A\mathrm{e}^{-3t}\cos\left(-2t\right)+B\mathrm{e}^{-3t}\sin\left(-2t\right)$ or $\left(x=\right)\ P\mathrm{e}^{(-3+2\mathrm{i})t}+Q\mathrm{e}^{(-3-2\mathrm{i})t}$ or $\left(x=\right)\ \mathrm{e}^{-3t}\left(P\mathrm{e}^{2it}+Q\mathrm{e}^{-2it}\right)$	Correct complementary function in any form, allow if the "$x=$" is missing or wrong and accept for this mark if the CF is given fully in terms of x instead of t.	**A1**
	PI: $\left\{x=\right\}\lambda\mathrm{e}^{-3t}$	Correct form for the particular integral selected. Must include $\lambda\mathrm{e}^{-3t}$ but accept with any extra terms that correctly disappear when coefficients found. Accept "PI=". If $\lambda\mathrm{e}^{pt}$ is used $p=-3$ must be seen later.	**B1**
	$\frac{\mathrm{d}x}{\mathrm{d}t}=-3\lambda\mathrm{e}^{-3t}\ ;\ \frac{\mathrm{d}^2x}{\mathrm{d}t^2}=9\lambda\mathrm{e}^{-3t}$ $\Rightarrow 9\lambda\mathrm{e}^{-3t}+6\left(-3\lambda\mathrm{e}^{-3t}\right)+13\lambda\mathrm{e}^{-3t}=8\mathrm{e}^{-3t}$	Differentiates a PI of **any** form twice (**provided** it has at least one constant and is a function of t) **and** substitutes into the equation. Allow only sign/coefficient errors only in the differentiation. Their PI must lead to non-zero derivatives.	**M1**
	$\Rightarrow 9\lambda-18\lambda+13\lambda=8\Rightarrow\lambda=...\ \left(2\right)$	Proceeds to find the value of the constant **following use of a PI of the correct form**. Any unnecessary extra terms in the PI must be found to be zero	**dM1**
	$x="\mathrm{e}^{-3t}\left(A\cos 2t+B\sin 2t\right)"+2\mathrm{e}^{-3t}$	Correct general solution ft on **their CF only** – any CF **provided** it has at least one constant and is in terms of t. **Must have $x=$ …** Do not allow if their CF is miscopied or mathematically changed	**A1ft**
	Work with a PI of the form $\lambda t\mathrm{e}^{-3t}$ is B0M1dM0A0 max even if $2\mathrm{e}^{-3t}$ is obtained. Only condone incorrect variables if they are recovered but refer to the note for the first A1.		**(6)**

Question Number	Scheme	Notes	Marks
6(b)	$x = \frac{1}{2}$ at $t = 0$ $\Rightarrow \frac{1}{2} = A + 2 \quad \left(\Rightarrow A = -\frac{3}{2}\right)$	Uses the initial condition for x in their GS to find a linear equation in one or two constants. Allow for GS = CF or CF + PI and the constant may come from the +PI	**M1**
	$x = \mathrm{e}^{-3t}\left(A\cos 2t + B\sin 2t\right) + 2\mathrm{e}^{-3t}$ $\frac{\mathrm{d}x}{\mathrm{d}t} = \mathrm{e}^{-3t}\left(-2A\sin 2t + 2B\cos 2t\right) - 3\mathrm{e}^{-3t}\left(A\cos 2t + B\sin 2t\right) - 6\mathrm{e}^{-3t}$ Uses the **product rule** to differentiate their real GS obtaining an expression in terms of t of the correct form for their GS (sign and coefficient errors only – so do not allow e.g., $...\mathrm{e}^{pt} \rightarrow ...\mathrm{e}^{qt}$). Allow for GS = CF or CF + PI and does not have to include constants. If they work with a complex function e.g., $x = P\mathrm{e}^{(-3+2\mathrm{i})t} + Q\mathrm{e}^{(-3-2\mathrm{i})t} + 2\mathrm{e}^{-3t}$ progress is unlikely. This mark is not scored for work in (c)		**M1**
	$t = 0, \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{2} \Rightarrow \frac{1}{2} = 2B - 3A - 6 \Rightarrow B = ...\left(= 1\right)$ Uses both initial conditions to find values for the 2 constants (no others) in their **GS = (CF with 2 constants) + PI(no constants)**. One constant must be found to be non-zero. **Requires both previous M marks.**		**ddM1**
	Examples: $x = \mathrm{e}^{-3t}\left(-\frac{3}{2}\cos 2t + \sin 2t\right) + 2\mathrm{e}^{-3t}$ or $x = \mathrm{e}^{-3t}\left(-\frac{3}{2}\cos 2t + \sin 2t + 2\right)$ or $x = 2\mathrm{e}^{-3t} - \frac{3}{2}\mathrm{e}^{-3t}\cos 2t + \mathrm{e}^{-3t}\sin 2t$	Correct particular solution in any form in terms of t. **Must be x = ...** <u>unless</u> this was the only reason for final A0 in part (a) due to omission or e.g, "y = ..." was used	**A1**
			(4)
(c)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \mathrm{e}^{-3t}\left(3\sin 2t + 2\cos 2t\right) - 3\mathrm{e}^{-3t}\left(-\frac{3}{2}\cos 2t + \sin 2t\right) - 6\mathrm{e}^{-3t} = 0$ Sets an expression for $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$. Accept with any unfound constants provided $\frac{\mathrm{d}x}{\mathrm{d}t} = \mathrm{f}(t)$		**M1**
	$\left(3\sin 2t + 2\cos 2t\right) - 3\left(-\frac{3}{2}\cos 2t + \sin 2t\right) - 6 = 0$ Achieves an equation of the form $a\sin bt + c\cos bt + d = 0$ <u>or equivalent with terms uncollected</u>. One of a and c non-zero and b and d non-zero. **Must follow a GS = CF + PI where two constants were found for the CF and one for the PI. Requires previous M mark**.		**dM1**
	$\cos 2t = \frac{12}{13} \Rightarrow t = 0.1973955598... \Rightarrow x$ or $a = \frac{1}{2}\mathrm{e}^{-3(0.1973...)}\left(4 - 3\times\frac{12}{13} + 2\sin(2\times 0.1973...)\right) = ..$ Finds a value of t from $\cos kt = c$ $\left(k \neq 1,\ -1 < c < 1\right)$ and uses their positive (or made positive) value of t to find a value of x (or a) via their PS. Accept a pair of stated values. **Requires both previous M marks.**		**ddM1**
	x or a = 0.553(1164729...)	awrt 0.553	**A1**
			(4)
			Total 14