6. The differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 13x = 8e^{-3t} \qquad t \geqslant 0$$

describes the motion of a particle along the x-axis.

(a) Determine the general solution of this differential equation.

(6)

Given that the motion of the particle satisfies $x = \frac{1}{2}$ and $\frac{dx}{dt} = \frac{1}{2}$ when $t = 0$

(b) determine the particular solution for the motion of the particle.

(4)

On the graph of the particular solution found in part (b), the first turning point for $t > 0$ occurs at $x = a$.

(c) Determine, to 3 significant figures, the value of a.

[*Solutions relying entirely on calculator technology are not acceptable.*]

(4)

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Question 6 continued

(Total for Question 6 is 14 marks)