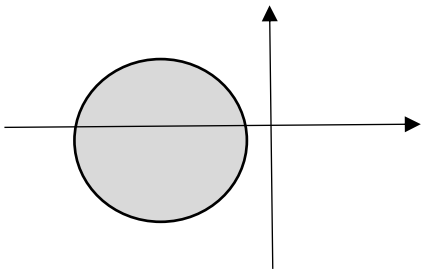
Question Number	Scheme	Notes	Marks
7(a) **Way 1**	$w=\dfrac{z-3}{2\mathrm{i}-z}\Rightarrow 2\mathrm{i}w-wz=z-3\Rightarrow z=...$	Attempts to make z the subject and obtains any f(w)	**M1**
	$z=\dfrac{3+2\mathrm{i}w}{w+1}$ or $\dfrac{-3-2\mathrm{i}w}{-w-1}$	Any correct expression for z in terms of w	**A1**
	$=\dfrac{3+2\mathrm{i}u-2v}{u+\mathrm{i}v+1}\times\dfrac{u+1-\mathrm{i}v}{u+1-\mathrm{i}v}$ Applies $w = u + \mathrm{i}v$ and a correct multiplier for their z seen or implied by a correct result from their z. Denominator must have had a "w". Note alternative route below.		**M1**
	$x+\mathrm{i}y=\dfrac{3+2\mathrm{i}u-2v}{u+\mathrm{i}v+1}\times\dfrac{u+1-\mathrm{i}v}{u+1-\mathrm{i}v}=\dfrac{(3-2v)(u+1)+2uv+2u(u+1)\mathrm{i}-(3-2v)v\mathrm{i}}{(u+1)^2+v^2}$ $y=x+3$ oe $\Rightarrow\dfrac{2u(u+1)-(3-2v)v}{(u+1)^2+v^2}=\dfrac{(3-2v)(u+1)+2uv}{(u+1)^2+v^2}+3$ Multiplies, extracts real and imaginary parts and uses them in the equation $y = x + 3$ (oe) to produce an equation in u and v only – no "i"s. Condone $y = ...\mathrm{i}$ if recovered. Can follow slips with multiplier but denominator of z must have had a "w" Note: Just $2u(u+1)-(3-2v)v=(3-2v)(u+1)+2uv+3$ is M0 (lost denominators)		**M1**
	$2u(u+1)-(3-2v)v=(3-2v)(u+1)+2uv+3(u+1)^2+3v^2$ $\Rightarrow u^2+7u+v^2+v+6=0$	Expands and simplifies to obtain an equation of a circle with 4 or 5 real unlike terms. **All previous Ms required.**	**dddM1**
	Alternative for the above 3 marks (note this could be done by equating expressions for y) $x+\mathrm{i}y=\dfrac{3+2\mathrm{i}u-2v}{u+\mathrm{i}v+1}\Rightarrow\Big(x+\mathrm{i}\big(x+3\big)\Big)\big(u+1+\mathrm{i}v\big)=3+2u\mathrm{i}-2v$ **M1**: Applies $z=x+\mathrm{i}y$, uses $y = x + 3$ and cross multiplies		
	$x(u+1)-v(x+3)+(x+3)(u+1)\mathrm{i}+xv\mathrm{i}=3-2v+2u\mathrm{i}$ $\Rightarrow ux+x-vx-3v=3-2v,\quad ux+x+3u+3+xv=2u$ $\Rightarrow x=\dfrac{3+v}{u+1-v},\quad x=\dfrac{-u-3}{u+1+v}$ **M1**: Equates real and imaginary parts and makes x the subject twice		
	$(3+v)(u+1+v)=-(u+3)(u+1-v)\Rightarrow 3u+3+3v+uv+v+v^2=-u^2-u+uv-3u-3+3v$ $\Rightarrow u^2+v^2+7u+v+6=0$ **M1**: Equates expressions for x to obtain a circle equation with 4 or 5 real unlike terms		
	$\Rightarrow\left(u+\dfrac{7}{2}\right)^2+\left(v+\dfrac{1}{2}\right)^2=\dfrac{49}{4}+\dfrac{1}{4}-6=\dfrac{13}{2}\Rightarrow$ centre: $\left(-\dfrac{7}{2},-\dfrac{1}{2}\right)$ radius: $\dfrac{\sqrt{26}}{2}$ or $\sqrt{\dfrac{13}{2}}$ **M1**: Extracts the centre **and/or** radius from their **circle** equation, however obtained, with 4 or 5 real unlike terms. Circle equation must not be in terms of z or w. They must get one correct coordinate (but condone wrong sign) or the correct radius for their circle. May use $u^2+v^2+2gu+2fv+c=0\Rightarrow$ centre: $(-g,-f)$, radius $=\sqrt{g^2+f^2-c}$ **A1**: For a correct centre or radius **from a correct circle equation** **A1**: For correct centre and radius **from a correct circle equation** Centre as coordinates, x/u=..., y/v =... or as $-\dfrac{7}{2}-\dfrac{1}{2}\mathrm{i}$ and allow $\left(-\frac{7}{2},-\frac{1}{2}\mathrm{i}\right)$ Allow exact equivalents for coordinates/radius		**M1** **A1** **A1**
			(8)

Question Number	Scheme	Notes	Marks
7(a) **Way 2**	$w=\dfrac{z-3}{2\mathrm{i}-z}=\dfrac{x+\mathrm{i}y-3}{2i-x-\mathrm{i}y}=\dfrac{x-3+\mathrm{i}\left(x+3\right)}{2\mathrm{i}-x-\mathrm{i}\left(x+3\right)}$ [Note that it is possible to replace x with $y-3$]	M1: Uses $z = x + \mathrm{i}y$ and $y = x + 3$ in the given transformation A1: Correct expression for w in terms of x	**M1** **A1**
	$\dfrac{x-3+\mathrm{i}\left(x+3\right)}{-x-\mathrm{i}\left(x+1\right)}=u+\mathrm{i}v\Rightarrow x-3+\mathrm{i}\left(x+3\right)=-xu+v\left(x+1\right)-\mathrm{i}u\left(x+1\right)-\mathrm{i}vx$	Applies $w = u + \mathrm{i}v$ and multiplies	**M1**
	$x-3=-ux+vx+v,\quad x+3=-ux-u-vx$ $x=\dfrac{3+v}{1+u-v},\quad x=\dfrac{-3-u}{1+u+v}$	Equates real and imaginary parts and makes x the subject twice	**M1**
	$3+3u+3v+v+uv+v^2=-3-3u+3v-u-u^2+uv$ $\Rightarrow u^2+v^2+7u+v+6=0$	Equates expressions for x to obtain a circle equation with 4 or 5 real unlike terms. **All previous Ms required.**	**dddM1**
	$\Rightarrow\left(u+\dfrac{7}{2}\right)^2+\left(v+\dfrac{1}{2}\right)^2=\dfrac{49}{4}+\dfrac{1}{4}-6=\dfrac{13}{2}\Rightarrow$ centre: $\left(-\dfrac{7}{2},-\dfrac{1}{2}\right)$ radius: $\dfrac{\sqrt{26}}{2}$ or $\sqrt{\dfrac{13}{2}}$ **M1**: Applies a correct process to extract the centre and/or radius from a circle equation, however obtained, with 4 or 5 real unlike terms. One correct coordinate (but condone wrong sign) or radius correct for their circle. May use $u^2+v^2+2gu+2fv+c=0\Rightarrow$ centre: $(-g,-f)$, radius $=\sqrt{g^2+f^2-c}$ **A1**: For correct centre or radius from a correct circle equation **A1**: For correct centre and radius from a correct circle equation Centre as coordinates, x/u=..., y/v =... or as $-\dfrac{7}{2}-\dfrac{1}{2}\mathrm{i}$ and allow $\left(-\frac{7}{2},-\frac{1}{2}\mathrm{i}\right)$ **(8)**		**M1** **A1** **A1**
Way 3	e.g., 3 points on line are (0,3), (1,4) and (2,5) or $z_1=3\mathrm{i},\ z_2=1+4\mathrm{i},\ z_3=2+5\mathrm{i}$	Attempts three points/complex numbers on $y = x + 3$ with 2 correct	**M1**
	$w=\dfrac{z-3}{2\mathrm{i}-z}\Rightarrow w_1=\dfrac{3\mathrm{i}-3}{-\mathrm{i}}\quad w_2=\dfrac{-2+4\mathrm{i}}{-1-2\mathrm{i}}\quad w_3=\dfrac{-1+5\mathrm{i}}{-2-3\mathrm{i}}$	Correct transformed complex numbers	**A1**
	$w_1=\dfrac{3\mathrm{i}-3}{-\mathrm{i}}\times\dfrac{\mathrm{i}}{\mathrm{i}}\quad w_2=\dfrac{-2+4\mathrm{i}}{-1-2\mathrm{i}}\times\dfrac{-1+2\mathrm{i}}{-1+2\mathrm{i}}\quad w_3=\dfrac{-1+5\mathrm{i}}{-2-3\mathrm{i}}\times\dfrac{-2+3\mathrm{i}}{-2+3\mathrm{i}}$ At least two correct multipliers to remove "i" from denominator seen or implied (one if (–1, 2) used). Requires 2 correct points/complex numbers on line		**M1**
	$w_1=-3-3\mathrm{i}\quad w_2=-\dfrac{6}{5}-\dfrac{8}{5}i\quad w_3=-1-\mathrm{i}$	Two correct complex numbers in $a + \mathrm{i}b$ form or as points	**M1**
	$6g+6f-c=18$ e.g., $x^2+y^2+2gx+2fy+c=0\Rightarrow \frac{12}{5}g+\frac{16}{5}f-c=0$ $2g+2f-c=0$	Uses a correct general equation of a circle to form three simultaneous equations. **All previous Ms required.**	**dddM1**
	$\Rightarrow g=\dfrac{7}{2},\ f=\dfrac{1}{2},\ c=6\Rightarrow$ centre $(-g,-f):\left(-\dfrac{7}{2},-\dfrac{1}{2}\right)$ radius $=\sqrt{g^2+f^2-c}=\dfrac{\sqrt{26}}{2}$ or $\sqrt{\dfrac{13}{2}}$ **M1**: Solves and obtains at least one correct coordinate (but condone wrong sign) or radius for their constants **A1**: Correct centre or radius from correct work **A1**: Correct centre and radius from correct work **(8)**		**M1** **A1** **A1**

Question Number	Scheme	Notes	Marks
7(b) (i) & (ii)		M1: **Any circle with the whole interior indicated**. Ignore any inconsistencies with their stated centre, value for radius (which may have been negative) or circle equation. If shaded, consider the shaded area but if not allow any credible indication such as an "*R*" inside the circle unless they have **clearly** indicated a segment. A1: **Correct circle drawn in the correct position with whole interior shaded**. Entirely in quadrants 2 & 3 and centre if marked in Q3 (if not marked then more than half of the circle in Q3). Condone if it appears that the area above the *x*-axis is greater than the area below provided the centre is indicated in Q3. **Must be shaded** but does not require a label. Circumference may be dotted/dashed line. Ignore incorrect labelling of centre/axes/intersections but **requires full marks in (a).**	**M1 (B1 on ePen)** **A1 (B1 on ePen)**
			(2)
			Total 10

Question Number	Scheme	Notes	Marks
8(a)	Allow "single fraction" to be implied by sum/difference of fractions with same denominator or a product of fractions. No further fractions in numerator/denominator.		