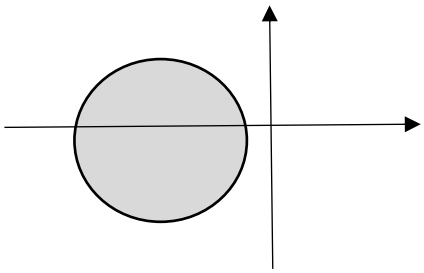
Question Number	Scheme	Notes	Marks
7(b) (i) & (ii)		M1: **Any circle with the whole interior indicated**. Ignore any inconsistencies with their stated centre, value for radius (which may have been negative) or circle equation. If shaded, consider the shaded area but if not allow any credible indication such as an "*R*" inside the circle unless they have **clearly** indicated a segment. A1: **Correct circle drawn in the correct position with whole interior shaded**. Entirely in quadrants 2 & 3 and centre if marked in Q3 (if not marked then more than half of the circle in Q3). Condone if it appears that the area above the *x*-axis is greater than the area below provided the centre is indicated in Q3. **Must be shaded** but does not require a label. Circumference may be dotted/dashed line. Ignore incorrect labelling of centre/axes/intersections but **requires full marks in (a).**	**M1 (B1 on ePen)** **A1 (B1 on ePen)**
			(2)
			Total 10

Question Number	Scheme	Notes	Marks
8(a)	Allow "single fraction" to be implied by sum/difference of fractions with same denominator or a product of fractions. No further fractions in numerator/denominator.		

	$\cot 2x\left\{+\tan x\right\}=\dfrac{\cos 2x}{\sin 2x}\left\{+\dfrac{\sin x}{\cos x}\right\}$	Uses $\cot 2x=\dfrac{\cos 2x}{\sin 2x}$ or e.g., $\dfrac{\cos 2x}{2\sin x\cos x}$	**M1**
	$\dfrac{\cos 2x+2\sin^2 x}{2\sin x\cos x}\Rightarrow$ e.g., $\dfrac{1-2\sin^2 x+2\sin^2 x}{2\sin x\cos x}$ or $\dfrac{\cos^2 x-\sin^2 x+2\sin^2 x}{2\sin x\cos x}$ $\dfrac{2\cos^2 x-1+2\sin^2 x}{\sin 2x}$ or $\dfrac{\cos 2x+1-\cos 2x}{\sin 2x}$ OR $\dfrac{\cos 2x+\tan x\sin 2x}{\sin 2x}\Rightarrow$ $\Rightarrow\dfrac{\cos 2x+\dfrac{\sin x}{\cos x}\times 2\sin x\cos x}{\sin 2x}\Rightarrow$ e.g., $\dfrac{1-2\sin^2 x+2\sin^2 x}{\sin 2x}$ OR $\dfrac{\cos 2x\cos x+\sin x\sin 2x}{\sin 2x\cos x}\Rightarrow$ $\dfrac{\cos x}{\sin 2x\cos x}$ or $\dfrac{\cos^3 x-\sin^2 x\cos x+2\sin^2 x\cos x}{\sin 2x\cos x}$	Uses sufficient correct identities e.g., $\cos 2x=1-2\sin^2 x$ $\cos 2x=\cos^2 x-\sin^2 x$ $\cos 2x=2\cos^2 x-1$ $2\sin^2 x=1-\cos 2x$ $\cos 2x\cos x+\sin x\sin 2x=\cos\left(2x-x\right)$ to obtain a correct single fraction with numerator in terms of sin x and/or cos x or "$\cos 2x+1-\cos 2x$". A qualifying fraction must be seen before $\dfrac{1}{2\sin x\cos x}$ or $\dfrac{1}{\sin 2x}$ Condone poor notation.	**A1 (M1 on ePen)**
	$=\dfrac{1}{2\sin x\cos x}$ or $\dfrac{1}{\sin 2x}=\text{cosec}\,2x*$	Fully correct proof with one of the two intermediate fractions seen. All notation correct – no mixed or missing arguments or e.g. sin x^2 for this mark.	**A1***
			(3)
Alt	$\cot 2x\left\{+\tan x\right\}=\dfrac{1-\tan^2 x}{2\tan x}\left\{+\tan x\right\}$	Uses $\cot 2x=\dfrac{1-\tan^2 x}{2\tan x}$	**M1**
	$\dfrac{1-\tan^2 x+2\tan^2 x}{2\tan x}\Rightarrow$ e.g., $\dfrac{\tan^2 x+1}{2\tan x}\Rightarrow\dfrac{\left(\frac{\sin x}{\cos x}\right)^2+1}{2\frac{\sin x}{\cos x}}\Rightarrow\dfrac{\cos x\left(\sin^2 x+\cos^2 x\right)}{2\cos^2 x\sin x}$ or $\dfrac{\tan^2 x+1}{2\tan x}\left\{\times\dfrac{\cos x}{\cos x}\right\}\Rightarrow\dfrac{\sin^2 x+\cos^2 x}{2\sin x\cos x}$ or $\dfrac{\sec^2 x}{2\tan x}$ or $\dfrac{\cos x}{2\cos^2 x\sin x}$	Uses correct identities e.g., $\tan x=\dfrac{\sin x}{\cos x}$ oe to obtain a correct single fraction in sin x and cos x but allow $\dfrac{\sec^2 x}{2\tan x}$ following use of $\sec^2 x=1+\tan^2 x$ A qualifying fraction must be seen before $\dfrac{1}{2\sin x\cos x}$ or $\dfrac{1}{\sin 2x}$ Condone poor notation.	**A1 (M1 on ePen)**
	$\dfrac{1}{2\sin x\cos x}$ or $\dfrac{1}{\sin 2x}=\text{cosec}\,2x*$	Fully correct proof with one of the two intermediate fractions seen. All notation correct – no mixed or missing arguments or e.g. sin x^2 for this mark.	**A1***
			(3)

Question Number	Scheme	Notes	Marks
8(b)	Examples:		**M1** **A1**

	$y^2 = w\sin 2x \Rightarrow 2y\frac{dy}{dx} = \frac{dw}{dx}\sin 2x + 2w\cos 2x$ or $y = w^{\frac{1}{2}}(\sin 2x)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}w^{\frac{1}{2}}(\sin 2x)^{-\frac{1}{2}}(2\cos 2x) + \frac{1}{2}w^{-\frac{1}{2}}\frac{dw}{dx}(\sin 2x)^{\frac{1}{2}}$ or $w = \frac{y^2}{\sin 2x} \Rightarrow \frac{dw}{dx} = \frac{2y\sin 2x\frac{dy}{dx} - y^2.2\cos 2x}{\sin^2 2x}$ or $w = y^2\text{cosec}2x \Rightarrow 2y\frac{dy}{dx}\text{cosec}2x - 2y^2\text{cosec}2x\cot 2x$ **M1**: Attempts the differentiation of the given substitution using the product/quotient and chain rules **and** obtains an **equation** in $\frac{dy}{dx}$ and $\frac{dw}{dx}$ of the correct form (sign/coefficient errors only and allow sign errors with quotient/product rule). This mark is not available for work in $\frac{dy}{dw}$ or $\frac{dw}{dy}$ unless appropriate work follows to achieve an equation in $\frac{dy}{dx}$ and $\frac{dw}{dx}$ of the correct form. **A1**: Correct differentiation	
	$y\frac{dy}{dx} + y^2\tan x = \sin x \rightarrow$ e.g., $\frac{1}{2}\left(\frac{dw}{dx}\sin 2x + 2w\cos 2x\right) + w\sin 2x\tan x = \sin x$ A recognisable attempt to eliminate y from the original equation to obtain an equation involving $\frac{dw}{dx}$, w and x only . Not dependent.	**M1**
	$\Rightarrow \frac{dw}{dx} + 2w\left(\cot 2x + \tan x\right) = \frac{2\sin x}{\sin 2x}$ $\Rightarrow \frac{dw}{dx} + 2w\text{cosec } 2x = \sec x$ * Fully correct work leading to the given equation with $2w\left(\cot 2x + \tan x\right)$ or e.g., $2w\cot 2x + 2w\tan x$ clearly replaced by $2w\text{cosec } 2x$ but allow cot 2x written as $\frac{1}{\tan 2x}$ or $\frac{\cos 2x}{\sin 2x}$ and/or tan x written as $\frac{\sin x}{\cos x}$ If the result in (a) is not clearly used there must be full equivalent work. Allow use of "csc 2x"	**A1***
		(4)

Question Number	Scheme	Notes	Marks

8(c)	$\frac{dw}{dx}+2w\operatorname{cosec}2x=\sec x \Rightarrow \text{IF}=e^{2\int \operatorname{cosec}2x\,dx}=\tan x$ or $e^{-\ln(\operatorname{cosec}2x+\cot 2x)} \Rightarrow \frac{1}{\operatorname{cosec}2x+\cot 2x}$ or $\frac{1}{\cot x}$ or $\tan x$	**M1**: $e^{2\int \operatorname{cosec}2x(dx)}$ condoning omission of one or both "2"s **A1**: $\tan x$ oe Allow $k\tan x$ e.g., $e^{2c}\tan x$ Not just $e^{\ln(\tan x)}$	**M1** **A1**
	$\Rightarrow w"\tan x"=\int "\tan x"\sec x\,\{dx\}$	Correctly applies their integrating factor to the equation, i.e., $\Rightarrow \text{IF}\times w=\int \text{IF}\times\sec x\,\{dx\}$ Allow equivalents for sec *x*. Condone "*y*" used for "*w*" for this mark.	**M1**
	$\Rightarrow w\tan x=\sec x(+c)$	Correct equation oe with or without constant.	**A1**
	Using $\text{IF}=\frac{1}{\operatorname{cosec}2x+\cot 2x} \Rightarrow$ RHS of $\int \frac{\sec x}{\operatorname{cosec}2x+\cot 2x}dx$ which is likely to need rewriting as $\int \tan x\sec x\,dx$ Note that IBP on $\sec x\tan x$ by writing it as $\sec^2 x\sin x$ can lead to $\sin x\tan x+\cos x(+c)$ Use Review for any attempts at integration you are unsure about.		
	e.g., $y^2=w\sin 2x$ and $w\tan x=\sec x+c \Rightarrow \frac{y^2}{\sin 2x}\tan x=\sec x+c$ $\Rightarrow y^2=\ldots \quad \left\{\frac{\sin 2x}{\tan x}(\sec x+c)\right\}$ Substitutes for *w* correctly and reaches $y^2=\ldots$ Their $y^2=\ldots$ must be consistent with **their** equation in *w* and *x* that immediately followed their integration. **This mark requires both previous M marks and an attempt at integration that includes a "+ *c*"** A further example is: $w=\operatorname{cosec}x+\frac{c}{\tan x} \Rightarrow y^2=\operatorname{cosec}x\sin 2x+\frac{c\sin 2x}{\tan x}$	**ddM1**
	$\left\{\text{e.g., } y^2=\frac{2\sin x\cos^2 x}{\sin x}\left(\frac{1}{\cos x}+c\right)\Rightarrow\right\}$ $y^2=2\cos x+A\cos^2 x$ Any correct $y^2=\ldots$ equation with RHS fully in terms of cos *x*. E.g. accept $y^2=2\cos x+2c\cos^2 x \quad y^2=\cos x(2+A\cos x) \quad y^2=2\cos^2 x\left(\frac{1}{\cos x}+c\right)$ Ignore any inconsistencies with the constant e.g., 2*c* later written as *c*	**A1**
			(6)
			Total 13

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