8. (a) For all the values of x where the identity is defined, prove that

$$\cot 2x + \tan x \equiv \operatorname{cosec} 2x$$

(3)

(b) Show that the substitution $y^2 = w \sin 2x$, where w is a function of x, transforms the differential equation

$$y\frac{\mathrm{d}y}{\mathrm{d}x} + y^2 \tan x = \sin x \qquad 0 < x < \frac{\pi}{2} \qquad \text{(I)}$$

into the differential equation

$$\frac{\mathrm{d}w}{\mathrm{d}x} + 2w \operatorname{cosec} 2x = \sec x \qquad 0 < x < \frac{\pi}{2} \qquad \text{(II)}$$

(4)

(c) By solving differential equation (II), determine a general solution of differential equation (I) in the form $y^2 = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is a function in terms of $\cos x$

$$\left[\text{You may use without proof } \int \operatorname{cosec} 2x \, \mathrm{d}x = \frac{1}{2} \ln \left| \tan x \right| \ (+\text{constant})\right]$$

(6)

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(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

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