Question Number	Scheme	Marks
2(a)	$x\dfrac{\mathrm{d}y}{\mathrm{d}x}-y^3=4\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}+x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-3y^2\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ Alt: $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{y^3}{x}+\dfrac{4}{x}\Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\dfrac{y^3}{x^2}+\dfrac{3y^2}{x}\dfrac{\mathrm{d}y}{\mathrm{d}x}-\dfrac{4}{x^2}$	M1A1
	$\Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+x\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}+\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-6y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2-3y^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$ Alt: $\Rightarrow \dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=\dfrac{2y^3}{x^3}-\dfrac{3y^2}{x^2}\dfrac{\mathrm{d}y}{\mathrm{d}x}-\dfrac{3y^2}{x^2}\dfrac{\mathrm{d}y}{\mathrm{d}x}+\dfrac{1}{x}\left(6y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2+3y^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)+\dfrac{8}{x^3}$	dM1
	$\Rightarrow x\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=6y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2+\left(3y^2-2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$	A1
		(4)
(b)	$x=2,\ y=1\Rightarrow \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=2}=\dfrac{4+1}{2}=\dfrac{5}{2},\ \left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_{x=2}=\dfrac{3(1)\left(\frac{5}{2}\right)-\frac{5}{2}}{2}=\dfrac{5}{2}$ $\left(\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)_{x=2}=\dfrac{6(1)\left(\frac{25}{4}\right)+\frac{5}{2}}{2}=20$	M1
	$(y=)\mathrm{f}(2)+(x-2)\mathrm{f}'(2)+\dfrac{(x-2)^2}{2}\mathrm{f}''(2)+\dfrac{(x-2)^3}{6}\mathrm{f}'''(2)+\ldots$ $=1+\dfrac{5}{2}(x-2)+\dfrac{5}{2}\dfrac{(x-2)^2}{2}+20\dfrac{(x-2)^3}{6}+\ldots$	M1
	$(y=)1+\dfrac{5}{2}(x-2)+\dfrac{5}{4}(x-2)^2+\dfrac{10}{3}(x-2)^3+\ldots$	A1
		(3)
		Total 7

Notes: Accept other derivative notations, e.g. *y*', throughout.

(a)

M1: $x\dfrac{\mathrm{d}y}{\mathrm{d}x}\to\dfrac{\mathrm{d}y}{\mathrm{d}x}+x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ or $y^3\to ky^2\dfrac{\mathrm{d}y}{\mathrm{d}x}$ Alternatively for $\dfrac{y^3}{x}\to\pm\alpha\dfrac{y^3}{x^2}+\dfrac{ky^2}{x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$ (oe via quotient rule), or any correct attempt at implicit differentiation on the equation (coefficient slips condoned).

A1: Any fully correct second derivative expression. The two most common are shown in the scheme.

dM1: Differentiates again using the product rule correctly at least once. Depends on the first method mark. Again, look for correct forms, but allow if coefficients are incorrect.

There may be alternatives where they substitute for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ before differentiating again.

The scheme follows the same principle. If unsure, use the Review system.

A1: Correct answer achieved from correct work. In the Alt they need to substitute the second derivative expression before reaching the final answer.

(b)

M1: Attempts to find values for the first 3 derivatives at $x = 2$. Working need not be seen, in which case accept if all are attempted with at least *y*' correct providing expressions involving the other derivatives were actually found in part (a).

M1: Correct application of Taylor's theorem with their values up to at least the $(x-2)^2$ term.

A1: Correct simplified expression. May be called *y* or f(*x*) or unlabelled for this part.