| Question Number | Scheme | Marks |
|---|---|---|
| **4(a)** | E.g. $2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{1}{t^2}\dfrac{\mathrm{d}t}{\mathrm{d}x}$ or $y^2=\dfrac{1}{t}\Rightarrow t=y^{-2}\Rightarrow\dfrac{\mathrm{d}t}{\mathrm{d}x}=-2y^{-3}\dfrac{\mathrm{d}y}{\mathrm{d}x}$ or $y=t^{-\frac{1}{2}}\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y}{\mathrm{d}t}\times\dfrac{\mathrm{d}t}{\mathrm{d}x}=-\dfrac{1}{2}t^{-\frac{3}{2}}\dfrac{\mathrm{d}t}{\mathrm{d}x}$ | B1 |
| | $\dfrac{\mathrm{d}y}{\mathrm{d}x}+y=xy^3\Rightarrow-\dfrac{t^{-\frac{3}{2}}}{2}\dfrac{\mathrm{d}t}{\mathrm{d}x}+\dfrac{1}{\sqrt{t}}=xt^{-\frac{3}{2}}$ | M1 |
| | $\dfrac{\mathrm{d}t}{\mathrm{d}x}-2t=-2x$ * | A1* |
| | | **(3)** |
| **(b)** | $I=\mathrm{e}^{-\int 2\,\mathrm{d}x}=\mathrm{e}^{-2x}$ | B1 |
| | $t\mathrm{e}^{-2x}=-2\int x\mathrm{e}^{-2x}\,\mathrm{d}x$ | M1 |
| | $=x\mathrm{e}^{-2x}-\int\mathrm{e}^{-2x}\,\mathrm{d}x$ | M1 |
| | $=x\mathrm{e}^{-2x}+\dfrac{1}{2}\mathrm{e}^{-2x}\left(+c\right)$ | A1 |
| | $t=\dfrac{1}{y^2}=x+\dfrac{1}{2}+c\mathrm{e}^{2x}$<br>$\Rightarrow y^2=\dfrac{1}{x+\frac{1}{2}+c\mathrm{e}^{2x}}\left(\text{or e.g.}\dfrac{2}{2x+1+A\mathrm{e}^{2x}}\right)$ (oe) | M1A1 |
| | | **(6)** |
| | | **Total 9** |

Notes:

(a)

B1: Any appropriate correct first derivative statement connecting $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}t}{\mathrm{d}x}$. May be implied by use of chain rule when substituting.

M1: Having attempted a first derivative statement, substitutes completely into the given differential equation to obtain an equation in $x$ and $t$ only (which may be the final step).

A1*: Achieves the printed answer with no errors.

(b)

B1: Correct integrating factor. May be implied by working.

M1: For $\text{their } IF\times t=\text{"}-2\text{"}\int x\times\text{their } IF\ \left(\mathrm{d}x\right)$ provided their IF is a function in $x$.

M1: Integrates RHS by parts to obtain $\alpha x\mathrm{e}^{\pm kx}+\beta\int\mathrm{e}^{\pm kx}\ \left(\mathrm{d}x\right)$

A1: Correct RHS (not follow through – must be the correct RHS). The + $c$ is not needed for this mark.

M1: Reverses the substitution and makes $y^2$ the subject. The constant of integration must have been introduced at the right time and been treated correctly, although the algebra in rearranging need not be fully correct.

A1: Correct answer in the required form with the constant correctly placed. Does not need to be fully simplified, and e.g. "2$c$" is fine for the constant and allow for "2$c$" renamed and "$c$" and such. ISW after a correct answer is seen.

| | | |
|---|---|---|
| **Alt (b)** | $m-2=0\Rightarrow m=2$ so CF is $t=A\mathrm{e}^{2x}$ | B1 |
| | For PI try $t=ax+b$ | M1 |
| | $\frac{\mathrm{d}t}{\mathrm{d}x}-2t=-2x\Rightarrow a-2(ax+b)=-2x\Rightarrow a=..,b=...$ | M1 |
| | $a=1,b=\frac{1}{2}\Rightarrow t=A\mathrm{e}^{2x}+x+\frac{1}{2}$ | A1 |
| | $t=\frac{1}{y^2}=x+\frac{1}{2}+A\mathrm{e}^{2x}$<br>$\Rightarrow y^2=\frac{1}{x+\frac{1}{2}+A\mathrm{e}^{2x}}\left(\text{or e.g.}\frac{2}{2x+1+B\mathrm{e}^{2x}}\right)$ (oe) | M1A1 |
| | | **(6)** |

Notes:

B1: Solves the auxiliary equation and forms the correct complementary function.

M1: Selects an appropriate form for the particular integral.

M1: Differentiates and substitutes into the differential equation to find values for the constants of the PI.

A1: Correct values and adds the CF and PI ($t$ = may be missing for this mark).

M1: Reverses the substitution and makes $y^2$ the subject. Must have formed an equation using **both** PI and CF for this mark.

A1: Correct answer in the required form with the constant correctly placed. Does not need to be fully simplified and same conditions for main scheme. ISW after a correct answer is seen.