Question Number	Scheme	Notes	Marks
6 Way 1	$w=\dfrac{z-\mathrm{i}}{z+1}\Rightarrow z=\dfrac{w+\mathrm{i}}{1-w}$ (oe)		B1
	$z=\dfrac{w+\mathrm{i}}{1-w}=\dfrac{u+\mathrm{i}v+\mathrm{i}}{1-u-\mathrm{i}v}\times\dfrac{1-u+\mathrm{i}v}{1-u+\mathrm{i}v}$		M1
	$z=\dfrac{u-u^2+uv\mathrm{i}-v\mathrm{i}-uv\mathrm{i}+\mathrm{i}-u\mathrm{i}-v^2-v+\mathrm{i}v+\mathrm{i}}{(1-u)^2+v^2}$ $\mathrm{Re}(z)=0\Rightarrow u-u^2-v^2-v=0$		M1A1
	$u-u^2-v^2-v=0\Rightarrow\left(u-\dfrac{1}{2}\right)^2+\left(v+\dfrac{1}{2}\right)^2=\dfrac{1}{2}$		dM1
	Centre : $\left(\dfrac{1}{2},-\dfrac{1}{2}\right)$ Radius : $\dfrac{1}{\sqrt{2}}$		A1A1
			(7)
Way 2	$w=\dfrac{z-\mathrm{i}}{z+1}\Rightarrow w=\dfrac{\mathrm{i}y-\mathrm{i}}{\mathrm{i}y+1}$		B1
	$w=\dfrac{\mathrm{i}y-\mathrm{i}}{\mathrm{i}y+1}\times\dfrac{1-\mathrm{i}y}{1-\mathrm{i}y}$		M1
	$u+\mathrm{i}v=\dfrac{\mathrm{i}y+y^2-\mathrm{i}-y}{1+y^2}=\dfrac{y^2-y}{1+y^2}+\dfrac{y-1}{1+y^2}\mathrm{i}\Rightarrow\dfrac{u}{v}=y$ $u=\dfrac{\frac{u^2}{v^2}-\frac{u}{v}}{1+\frac{u^2}{v^2}}$ or $v=\dfrac{\frac{u}{v}-1}{1+\frac{u^2}{v^2}}$ $\left(u-u^2-v^2-v=0\right)$		M1A1
	$u-u^2-v^2-v=0\Rightarrow\left(u-\dfrac{1}{2}\right)^2+\left(v+\dfrac{1}{2}\right)^2=\dfrac{1}{2}$		dM1
	Centre : $\left(\dfrac{1}{2},-\dfrac{1}{2}\right)$ Radius : $\dfrac{1}{\sqrt{2}}$		A1A1

Notes:

B1: Correct rearrangement as shown or equivalent.

M1: Introduces $w = u + \mathrm{i}v$ (or may use other variables) and multiplies numerator (and denominator) by the conjugate of the denominator.

M1: Expands the numerator and sets the real part to zero – allow if there are slips in the denominator e.g. wrong sign.

A1: Correct equation in any form.

dM1: Completes the square for u and v. They must have attained an expression on which they need to complete the square on at least on of u or v. Alternatively allow for correct centre up to sign error, or correct radius, deduced from expanded circle equation.

A1: Correct centre – but must be from a circle equation. Accept as coordinates or as $\dfrac{1}{2}-\dfrac{1}{2}\mathrm{i}$ and be tolerant of notation errors, e.g. $\left(\dfrac{1}{2},-\dfrac{1}{2}\mathrm{i}\right)$ if the correct point is clear.

A1: Correct radius following a correct equation in u and v (allowing sign slips on u and v). Must be exact, but accept equivalents.

Alternative:

B1: Replaces z with iy (oe), or may replaces by x +iy and later set $x = 0$ in the equation.

M1: Multiplies numerator and denominator by the conjugate of the denominator.

M1: Rearranges into real and imaginary parts to obtain y in terms of u and v and substitutes into u or v to eliminate y. Allow recovery from slips in the denominator per main scheme.

A1: Correct equation in any form.

dM1: Completes the square for u and v. They must have attained an expression on which they need to complete the square on at least on of u or v. Alternative as per main scheme.

A1: Correct centre. Accept as coordinates or as $\frac{1}{2}-\frac{1}{2}\text{i}$ and be tolerant of notation errors, e.g. $\left(\frac{1}{2},-\frac{1}{2}\text{i}\right)$ if the correct point is clear.

A1: Correct radius from correct work (allowing sign slips on u and v). Must be exact, but accept equivalents.

Note: Some rarer alternatives may be seen, but if you are unsure of a method send to review.

Way 3	$0 \to -\text{i}, \text{i} \to 0, \infty \to 1$	B1
	E.g. Bisector of 0 and 1 is $u = \frac{1}{2}$	M1
	Bisector of 0 and $-$i is $v = -\frac{1}{2}$	M1A1
	Bisectors meet at when $u = \frac{1}{2}$ and $v = -\frac{1}{2}$	dM1
	Centre : $\left(\frac{1}{2}, -\frac{1}{2}\right)$ Radius : $\frac{1}{\sqrt{2}}$	A1A1

Notes:

B1: Identifies three correct image points on the circle.

M1: Finds at least two image points on the circle and attempts the perpendicular bisector of at least one pair of points. Alt: attempts to substitute into a general circle equation and makes some progress eliminating one variable.

M1: Finds three image points and attempts two perpendicular bisectors. Alt: uses the third point to form a second equation in the same two variables.

A1: Correct bisectors. Alt: correct equations.

dM1: Finds the intersection of the bisectors. Note MMAM may be implied in this method if the bisectors are obvious (as with the three points shown in the scheme). Alt: valid method for fully solving the simultaneous equations.

A1: Correct centre. Accept as coordinates or as $\frac{1}{2}-\frac{1}{2}\text{i}$ and be tolerant of notation errors, e.g. $\left(\frac{1}{2},-\frac{1}{2}\text{i}\right)$ if the correct point is clear.

A1: Correct radius from correct work (allowing sign slips on u and v). Must be exact, but accept equivalents.

Way 4: Using inverse points of the imaginary axis, e.g. ± 1 is also possible. Here as -1 maps to ∞ so 1 maps to the centre of the circle, and then any other point may be used to work out the radius of the circle. If other inverse points are used it will be more complicated. Send to review if you see any responses like this that you feel deserve merit.