Question Number	Scheme	Marks
8(a)	E.g. $\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=\frac{1}{x}\frac{dy}{dt}$ or $t=\ln x\Rightarrow x=e^t\Rightarrow\frac{dx}{dy}=e^t\frac{dt}{dy}$ or $t=\ln x\Rightarrow\frac{dt}{dy}=\frac{1}{x}\frac{dx}{dy}$ etc.	B1
	E.g. $\frac{d^2y}{dx^2}=-\frac{1}{x^2}\frac{dy}{dt}+\frac{1}{x}\frac{d^2y}{dt^2}\frac{dt}{dx}$ or $\frac{dx}{dy}=e^t\frac{dt}{dy}\Rightarrow\frac{dy}{dx}=e^{-t}\frac{dy}{dt}\Rightarrow\frac{d^2y}{dx^2}=-e^{-t}\frac{dt}{dx}\frac{dy}{dt}+e^{-t}\frac{d^2y}{dt^2}\frac{dt}{dx}$ or $x\frac{dy}{dx}=\frac{dy}{dt}\Rightarrow x\frac{d^2y}{dx^2}+\frac{dy}{dx}=\frac{d^2y}{dt^2}\frac{dt}{dx}$ etc	M1
	E.g. $\left(t=\ln x\Rightarrow\frac{dt}{dx}=\frac{1}{x}\Rightarrow\right)\frac{d^2y}{dx^2}=-\frac{1}{x^2}\frac{dy}{dt}+\frac{1}{x}\frac{d^2y}{dt^2}\frac{dt}{dx}=-e^{-2t}\frac{dy}{dt}+e^{-2t}\frac{d^2y}{dt^2}$ or $\left(x=e^t\Rightarrow\frac{dx}{dt}=e^t\Rightarrow\right)\frac{d^2y}{dx^2}=-e^{-t}\frac{dt}{dx}\frac{dy}{dt}+e^{-t}\frac{d^2y}{dt^2}\frac{dt}{dx}=-e^{-2t}\frac{dy}{dt}+e^{-2t}\frac{d^2y}{dt^2}$ or $x\frac{d^2y}{dx^2}+\frac{dy}{dx}=\frac{d^2y}{dt^2}\frac{dt}{dx}\Rightarrow e^t\frac{d^2y}{dx^2}+\frac{1}{e^t}\frac{dy}{dt}=\frac{1}{e^t}\frac{d^2y}{dt^2}$ $\Rightarrow\frac{d^2y}{dx^2}=e^{-2t}\left(\frac{d^2y}{dt^2}-\frac{dy}{dt}\right)$*	A1*
		(3)
(b)	$x^2\frac{d^2y}{dx^2}-2y=1+4\ln x-2(\ln x)^2\Rightarrow e^{2t}\times e^{-2t}\left(\frac{d^2y}{dt^2}-\frac{dy}{dt}\right)-2y=1+4t-2t^2$ $\Rightarrow\frac{d^2y}{dt^2}-\frac{dy}{dt}-2y=1+4t-2t^2$ *	B1*
		(1)
(c)	$m^2-m-2=0\Rightarrow m=2,-1$ $y=Ae^{2t}+Be^{-t}$	M1 A1
	$y=at^2+bt+c\Rightarrow\frac{dy}{dt}=2at+b\Rightarrow\frac{d^2y}{dt^2}=2a$	M1
	$\Rightarrow 2a-2at-b-2at^2-2bt-2c=1+4t-2t^2$ $2a=2\Rightarrow a=1$ $-2a-2b=4\Rightarrow b=-3$ $2a-b-2c=1\Rightarrow c=2$	dM1
	$y=Ae^{2t}+Be^{-t}+t^2-3t+2$	A1
		(5)
(d)	$y=Ax^2+Bx^{-1}+(\ln x)^2-3\ln x+2$	B1ft
		(1)
		Total 10

Notes:

(a)

B1: Any correct equation linking $\frac{dy}{dx}$ and $\frac{dy}{dt}$ (or their reciprocals).

M1: Differentiates again wrt x or t with attempts at both the product rule **and** chain rule to obtain an equation involving $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{dt^2}$. Allow slips as long as an attempt at both rules is made.

A1*: Eliminates the x's and $\frac{dx}{dt}$'s from the equation and completes to obtain the printed answer with no errors.

(b)

B1*: Shows a clear substitution into the equation and proceeds to the given answer with no incorrect steps seen. Working may be minimal as long as there is at least one step before the given answer that shows substitution has been made.

(c)

M1: Forms and attempts (need not be a correct attempt) to solve $m^2 - m - 2 = 0$ and attempts to form the appropriate complementary function for their roots. Allow if variables are incorrect for the methods (ie x used instead of t).

A1: Correct complementary function. The $y =$… may be missing. Must be in terms of t.

M1: Starts with the correct form the particular integral and differentiates twice.

dM1: Substitutes into the correct differential equation and solves to find at least two of a and b and c by comparing coefficients. Do not be concerned about the method of solving, look for any attempt reaching values for at least two of the constants.

A1: Correct solution. Must start $y =$ … . Allow if stated in (d).

(d)

B1ft: Correct equation with exponentials simplified to powers of x (follow through their answer to (c)) and isw if incorrect simplification takes place after a correct answer seen. Should start $y =$ … accept if GS =, or similar, is used for **both** (c) and (d). Accept $\ln^2 x$ for $(\ln x)^2$ but not $\ln x^2$.

Note: parts (c) and (d) may be merged, but to award the A mark for (d) a correct general solution must be given. Allow the mark in (d) if seen in part (c).