8. (a) Given that $t = \ln x$, where $x > 0$, show that

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \mathrm{e}^{-2t}\left(\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - \frac{\mathrm{d}y}{\mathrm{d}t}\right)$$

(3)

(b) Hence show that the transformation $t = \ln x$, where $x > 0$, transforms the differential equation

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2y = 1 + 4\ln x - 2(\ln x)^2 \qquad \text{(I)}$$

into the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - \frac{\mathrm{d}y}{\mathrm{d}t} - 2y = 1 + 4t - 2t^2 \qquad \text{(II)}$$

(1)

(c) Solve differential equation (II) to determine y in terms of t.

(5)

(d) Hence determine the general solution of differential equation (I).

(1)

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