Question Number	Scheme	Marks
9(a)	$(\cos 6\theta =)\mathrm{Re}(\cos\theta + \mathrm{i}\sin\theta)^6 = \cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta$	M1A1
	$\cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta$ $= \cos^6\theta - 15\cos^4\theta(1-\cos^2\theta) + 15\cos^2\theta(1-\cos^2\theta)^2 - (1-\cos^2\theta)^3$	M1
	$= \cos^6\theta - 15\cos^4\theta + 15\cos^6\theta + 15\cos^2\theta(1-2\cos^2\theta+\cos^4\theta) - (1-3\cos^2\theta+3\cos^4\theta-\cos^6\theta)$ $\cos 6\theta \equiv 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$*	A1*
		(4)
(b)	$48x^6 - 72x^4 + 27x^2 - 1 = 0$ $x = \cos\theta \Rightarrow \frac{3}{2}\cos 6\theta + \frac{1}{2} = 0 \Rightarrow \cos 6\theta = \ldots\left(-\frac{1}{3}\right)$	M1
	$\cos 6\theta = -\frac{1}{3} \Rightarrow 6\theta = \ldots \Rightarrow \theta = \ldots$ (pv 6θ =109.5°, 1.91 rad, θ = 18.25°, 0.318 rad)	dM1
	$x = \cos\theta = \ldots$ (commonly 0.950)	dM1
	$x = 0.204$	A1
		(4)
		Total 8

Notes:

(a)

M1: Attempts the binomial expansion of $(\cos\theta + \mathrm{i}\sin\theta)^6$ and extracts the real terms. Binomial coefficients should be correct but allow if there are sign errors. Ignore errors on imaginary terms. Allow use of $(c+\mathrm{i}s)^6$ notation.

A1: Correct extracted expression. Need not be set equal to cos 6θ at this stage.

M1: Applies $\sin^2\theta = 1 - \cos^2\theta$ to obtain an expression in cos θ only.

A1*: Sets equal to cos 6θ and fully expands the square and cube brackets (must be shown) then completes with no errors to obtain the given answer.

(b)

Note: The question says hence and thus methods via solving the cubic in x^2 and square rooting the smallest root score no marks.

M1: Attempts to use the result from part (a) and obtains $\cos 6\theta = k,\ |k| < 1,\ k \neq 0$

dM1: Solves to obtain at least one value for θ. If the $6\theta = \arccos\left(-\frac{1}{3}\right)$ is seen, accept any value that follows for the attempt, but if this method is not shown answers must be correct for their value of $\cos(6\theta)$. The principal values are shown in the scheme. Other values that may be seen are

In radians: for 6θ 4.33, 8.19, 10.66, 14.48, 16.94 ; for θ 0.73, 1.37, 1.78, 2.41, 2.82

In degrees: for 6θ 251°, 469°, 611°, 829°, 1331°; for θ 41.8°, 78.2°, 102°, 138°, 222°;

dM1: Depends on first M. Reverses the substitution to find at least one value for x.

Full list of values for reference: 0.950, 0.746, 0.204, −0.204, −0.746, −0.950

A1: Awrt 0.204 and must be seen identified as the answer, not just given in a list.

Note those who find $\cos 6\theta = -\frac{1}{3}$ but go on to find $6\theta = \arccos\frac{1}{3} = 1.23$ (70.5°), and so $\theta = 0.205\ (11.8°)$ etc (0.979 as primary answer) may score the M1M0M1A0 but watch out for those who use $\pi - \arccos\frac{1}{3}$ as these can score full marks.

9(a) Alt	$\left(z+\frac{1}{z}\right)^6 = z^6+6z^4+15z^2+20+\frac{15}{z^2}+\frac{6}{z^4}+\frac{1}{z^6}$ $=\left(z^6+\frac{1}{z^6}\right)+6\left(z^4+\frac{1}{z^4}\right)+15\left(z^2+\frac{1}{z^2}\right)+20$ $=2\cos 6\theta+6\left(2\cos 4\theta\right)+15\left(2\cos 2\theta\right)+20$	M1A1
	$=2\cos 6\theta+12\left(2\cos^2 2\theta-1\right)+30\left(2\cos^2\theta-1\right)+20$ $=2\cos 6\theta+12\left(2\left(2\cos^2\theta-1\right)^2-1\right)+30\left(2\cos^2\theta-1\right)+20$	M1
	$64\cos^6\theta = 2\cos 6\theta+12\left(8\cos^4\theta-8\cos^2\theta+1\right)+60\cos^2\theta-30+20$ $\Rightarrow \cos 6\theta \equiv 32\cos^6\theta-48\cos^4\theta+18\cos^2\theta-1*$	A1*
		(4)

Notes:
(a)

M1: Attempts the binomial expansion of $\left(z+z^{-1}\right)^6$, groups terms (may be implied) and attempts to replace the z^n+z^{-n} to achieve an expression in cosines. Allow if there are errors in the "$2\cos(n\theta)$".

A1: Correct expression in cosines. Need not be set equal to $64\cos^6\theta$ at this stage.

M1: Applies $\cos 2A = 2\cos^2 A-1$ repeatedly to write the $\cos 4\theta$ and $\cos 2\theta$ in terms of $\cos\theta$ only.

A1*: Sets equal to $64\cos^6\theta$, expands all brackets and rearranges to the given expression with no errors to obtain the given answer.