Question	Scheme	Marks
1(a)	$\frac{dy}{dx} - 3y\tan x = \sec^2 x$ with $y = 4$ when $x = \frac{\pi}{4}$	
	Mark parts (a) and (b) together	
	$\text{IF} = \exp\left(\int -3\tan x\,dx\right) = \ldots$	**M1**
	$= \exp(3\ln\cos x) = \cos^3 x$ *	**A1***
		(2)
Alt (a)	$\frac{d}{dx}\left(y\cos^3 x\right) = \frac{dy}{dx}\cos^3 x + y\times 3\cos^2 x\times -\sin x$	**M1**
	$= \cos^3 x\left(\frac{dy}{dx} - 3y\tan x\right)$ hence integrating factor	**A1***
		(2)
(b)	$\Rightarrow y\cos^3 x = \int \sec^2 x\times\cos^3 x\,dx$	**M1**
	$\Rightarrow y\cos^3 x = \sin x\ \ (+c)$ o.e. **OR** $y = \sin x\sec^3 x + (c\sec^3 x)$ **OR** $y = \tan x\sec^2 x + (c\sec^3 x)$ o.e.	**A1**
	$\Rightarrow 4\cos^3\frac{\pi}{4} = \sin\frac{\pi}{4} + c \Rightarrow c = \frac{\sqrt{2}}{2} \Rightarrow y = \tan x\sec^2 x + \frac{\sqrt{2}}{2}\sec^3 x$ o.e.	**M1** **A1**
		(4)
		(6 marks)

Notes:

(a)

M1: Attempts the integrating factor directly. Allow for an attempt at $\exp\left(\pm\int k\tan x\right)$ but no need to evaluate the integral for this mark.

Must be shown, but could be implied by an integrated expression of the correct form e.g. $\exp(-3\ln\sec x)$ or $\exp(3\ln\cos x)$.

If $(\sec x)^{-3}$ is all that is seen then accept for M1 only

A1*: Correct simplified integrating factor. Correct processing seen, so exp and ln inverse applied correctly, and correct integration of $-3\tan x \longrightarrow -3\ln\sec x$ *or* $3\ln\cos x$ seen. **No sight of exp and ln being correctly processed, or no evidence of correct integration is max M1A0**

Alt by verification

M1: Attempts to differentiate $y\cos^3 x \to \cos^3 x\dfrac{\mathrm{d}y}{\mathrm{d}x} \pm ky\sin x\cos^2 x$

A1*: Shows correctly that the derivative is $\cos^3 x$ times equation (I) **and makes minimal conclusion.**

(b)

M1: Applies the integrating factor. Look for $y \times$ their IF $= \int \sec^2 x \times$ their IF $\mathrm{d}x$ or equivalent

A1: Correct general form. The constant may be missing for this mark.

M1: Applies the initial condition to find a value or c and treats it correctly (i.e. having the correct position, and not just appearing in isolation). Incorrect treatment of c is M0 (e.g. $y = \tan x\sec^2 x + c \to c = ...$ is M0)

A1: Correct answer. Accept alternatives in terms of cos x etc that are in the form $y =$ … so e.g.

$$y = \frac{\sin x}{\cos^3 x} + \frac{\sqrt{2}}{2\cos^3 x}$$

etc

Note: c could be given as $\frac{1}{\sqrt{2}}$. ISW once a correct answer is seen.