Question	Scheme	Marks
5(a)	$y = r\sin\theta = \left(\sqrt{3} + \dfrac{1-\cos\theta}{\sin\theta}\right)\sin\theta = \sqrt{3}\sin\theta + 1 - \cos\theta$ $\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}\theta} = \sqrt{3}\cos\theta + \sin\theta$ or $\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = \sqrt{3}\cos\theta + \cos\theta\tan\dfrac{\theta}{2} + \sin\theta\dfrac{1}{2}\sec^2\dfrac{\theta}{2}$ Or equivalent **NOTE** that $\sin\theta \times \frac{1}{2}\sec^2\frac{\theta}{2} \equiv \tan\frac{\theta}{2} \equiv \frac{1-\cos\theta}{\sin\theta}$ so final term may be written this way	**M1** **A1**
	$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = 0 \Rightarrow \sin\theta = -\sqrt{3}\cos\theta \Rightarrow \tan\theta = ... \Rightarrow \theta = ..$	**M1**
	$\theta\left(= \arctan\left(-\sqrt{3}\right)\right) = \dfrac{2\pi}{3}$	**A1**
		(4)
Alt (i) **For the 2nd M1A1**	$\sqrt{3}\cos\theta + \cos\theta\dfrac{1-\cos\theta}{\sin\theta} + \dfrac{1}{2}\sin\theta\left(1 + \dfrac{(1-\cos\theta)^2}{\sin^2\theta}\right) = 0$ $\Rightarrow 2\sqrt{3}\cos\theta\sin\theta + 2\cos\theta - 2\cos^2\theta + (\sin^2\theta + 1 - 2\cos\theta + \cos^2\theta) = 0$ $\Rightarrow \sqrt{3}\sin 2\theta - \cos 2\theta + 1 = 0$ $\Rightarrow 2\sin\left(2\theta - \dfrac{\pi}{6}\right) = -1 \Rightarrow \theta = ...$	
Alt (ii) **For the 2nd M1A1**	$\sqrt{3}\cos\theta + \cos\theta\tan\dfrac{\theta}{2} + \sin\theta\dfrac{1}{2}\sec^2\dfrac{\theta}{2} = 0 \Rightarrow 2\sqrt{3} + 2\tan\dfrac{\theta}{2} + \tan\theta\left(1 + \tan^2\dfrac{\theta}{2}\right) = 0$ $\Rightarrow 2\sqrt{3} + 2\tan\dfrac{\theta}{2} + \dfrac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\left(1 + \tan^2\dfrac{\theta}{2}\right) = 0 \Rightarrow \sqrt{3} + 2\tan\dfrac{\theta}{2} - \sqrt{3}\tan^2\dfrac{\theta}{2} = 0$ $\Rightarrow \theta = 2\arctan\sqrt{3} = ..$	
(b)	$\displaystyle\int r^2\mathrm{d}\theta = \int 3 + 2\sqrt{3}\tan\frac{\theta}{2} + \tan^2\frac{\theta}{2}\mathrm{d}\theta$	**M1**
	$\displaystyle= \int 3 + 2\sqrt{3}\tan\frac{\theta}{2} + \sec^2\frac{\theta}{2} - 1\ \mathrm{d}\theta$	**M1**
	$= 3\theta + 2\sqrt{3} \times -2\ln\cos\dfrac{\theta}{2} + 2\tan\dfrac{\theta}{2} - \theta$ (+c) o.e.	**M1** **A1**
	Substitutes their limits to reach Area $= \left(\dfrac{1}{2}\right)\left[2\dfrac{2\pi}{3} - 4\sqrt{3}\ln\cos\dfrac{\pi}{3} + 2\tan\dfrac{\pi}{3} - 0\right]$ o.e.	**M1**

	$= 2\sqrt{3}\ln 2 + \frac{2\pi}{3} + \sqrt{3}$	**A1**
		(6)
		(10 marks)

Notes:

(a)

M1: Applies $y = r\sin\theta$ and attempts the differentiation. The given identity may be applied first, or later, or not at all. Condone sign/coefficient errors only. Use of $y = r\cos\theta$ scores 0 marks in (a)

A1: Correct derivative.

M1: Sets the derivative equal to zero and solves for θ. If identity was used initially expect as scheme but alternative routes are possible, e.g. see examples above. Allow for any plausible attempt that reaches a value of θ if there are minor errors but attempts at trig identities should be correct up to sign error.

A1: Correct value- must be in correct quadrant. If not explicitly written in (a) but used in (b) as their limit then allow this mark to score in (b)

(b)

M1: Shows intention to find $\int r^2$ by expanding r^2 to obtain three terms in the integrand. There may be errors in the coefficients when squaring, but three terms should be achieved.

M1: Applies $\tan^2 A = \pm 1 \pm \sec^2 A$ to their integral. Not dependent so may be scored as long as they have a $\tan^2$ term. Note: some candidates may use the identity given in the question, but they must progress to terms which can be integrated (see Alt).

M1: Attempts the integration including $\sec^2 ... \to k\tan ...$ and $\tan ... \to p\ln\cos ...$ or equivalent (e.g. $p\ln\sec ...$). Some candidates may attempt to use the given identity to integrate the $\tan^2\frac{\theta}{2}$ term (see Alt), but there must be a complete method shown to get the correct form for the integral. Not dependent but must have achieved appropriate tan and $\sec^2$ terms, or equivalent terms that can be integrated (see Alt).

A1: Fully correct integration (though the $\frac{1}{2}$ and limits may be missing for this mark). **Note:** equivalent integration is possible using the identity given in (a) (see Alt).

M1: Applies their limits to their integral. Must see evidence of substitution into their integrated expression. Must have had appropriate tan and $\sec^2$ terms in their integral

A1: Correct answer in the form specified.

NOTE: If a mix of variables is seen, and later corrected prior to substitution of limits, then full marks can still be achieved, but if not recovered then deduct the penultimate A mark

Note: Omission of the ½ could result in M1M1M1A1M1A0

(b) **Alt for first 5 marks**	$\int r^2 \mathrm{d}\theta = \int 3 + 2\sqrt{3}\tan\frac{\theta}{2} + \tan^2\frac{\theta}{2}\mathrm{d}\theta$	**M1**
	$\int 3 + 2\sqrt{3}\tan\frac{\theta}{2} + \frac{1 - 2\cos\theta + \cos^2\theta}{\sin^2\theta}\ d\theta$ $= \int 3 + 2\sqrt{3}\tan\frac{\theta}{2} + \mathrm{cosec}^2\theta - 2\mathrm{cosec}\theta\cot\theta + \mathrm{cosec}^2\theta - 1\ d\theta$	**M1**
	$= 2\theta + 4\sqrt{3}\ln\sec\frac{\theta}{2} - 2\cot\theta + 2\,\mathrm{cosec}\theta\ (+c)$ o.e.	**M1** **A1**
	Substitutes their limits to reach Area $= \left(\frac{1}{2}\right)\left[2\frac{2\pi}{3} + 4\sqrt{3}\ln\sec\frac{\pi}{3} - 2\cot\frac{2\pi}{3} + 2\mathrm{cosec}\frac{2\pi}{3} - \cdots\right]$ o.e.	**M1**

Notes:

M1: Shows intention to find $\int r^2$ by expanding r^2 to obtain three terms in the integrand. There may be errors in the coefficients when squaring, but three terms should be achieved.

M1: Applies the given identity to their $\tan^2\frac{\theta}{2}$ term in the integral, separates terms and applies $\cot^2\theta = \pm\mathrm{cosec}^2\theta \pm 1$. Not dependent so may be scored as long as they have a $\tan^2$ term.

M1: Attempts the integration including $\mathrm{cosec}^2 \ldots \rightarrow k\cot \ldots \mathrm{cosec} \ldots \cot \ldots \longrightarrow k\mathrm{cosec}\theta$ and $\tan \ldots \rightarrow p\ln\cos \ldots$ or equivalent (e.g. $p\ln\sec \ldots$). there must be a complete method shown to get the correct form for the integral. Not dependent but must have achieved appropriate terms that can be integrated.

A1: Fully correct integration (though the $\frac{1}{2}$ and limits may be missing for this mark).

M1: Applies their limits to their integral. Must see evidence of substitution into their integrated expression. Must have had appropriate $\mathrm{cosec}^2\theta$ *and* $\mathrm{cosec}\theta\cot\theta$ terms in their integral

Note: If this method is used it is unlikely that the final A mark is scored, as substitution of the lower limit leads to an indeterminate form. However, this can be resolved by the appropriate treatment of limits e.g. L'Hopital's rule etc.