Question	Scheme	Marks
6(a)	$4\frac{d^2y}{dx^2}-4\frac{dy}{dx}+37y=6e^{5x}$	
	Aux. Eqn. $4m^2-4m+37=0\Rightarrow m=\frac{4\pm\sqrt{16-16\times37}}{8}=\frac{1}{2}\pm3i$	**M1**
	C.F. is $(y=)e^{\frac{1}{2}x}(A\cos3x+B\sin3x)$	**A1**
	PI try $(y=)\ ke^{5x}$	**B1**
	$\frac{dy}{dx}=5ke^{5x},\frac{d^2y}{dx^2}=25ke^{5x}\Rightarrow4(25ke^{5x})-4(5ke^{5x})+37(ke^{5x})=6e^{5x}$	**M1**
	$\Rightarrow 100k-20k+37k=6\Rightarrow k=\cdots\ \ (k=\frac{2}{39}or\ \frac{6}{117}\ o.e.)$	**M1**
	So gen sol. is $y=\frac{2}{39}e^{5x}+e^{\frac{1}{2}x}(A\cos3x+B\sin3x)$	**A1ft**
		(6)
(b)	$x=0,y=0\Rightarrow0=\frac{2}{39}+A$	**M1**
	$\frac{dy}{dx}=\frac{10}{39}e^{5x}+\frac{1}{2}e^{\frac{1}{2}x}(A\cos3x+B\sin3x)+e^{\frac{1}{2}x}(-3A\sin3x+3B\cos3x)$	**M1A1ft**
	$x=0,\frac{dy}{dx}=0\Rightarrow\frac{10}{39}+\frac{1}{2}A+3B=0\Rightarrow B=...,A=...$	**M1**
	$(y=)\ \frac{2}{39}e^{5x}+e^{\frac{1}{2}x}\left(-\frac{2}{39}cos\,3x-\frac{1}{13}sin\,3x\right)$	**A1**
		(5)
		(11 marks)

Notes:

(a)

M1: Forms correct auxiliary equation and attempts to solve by any valid means. Implied by correct roots. Usual rules for solving a quadratic apply.

A1: Correct complementary function, allow without the "$y=$". Accept $y=Ae^{\left(\frac{1}{2}+3i\right)x}+Be^{\left(\frac{1}{2}-3i\right)x}$ or from De Moivre can use e.g. $y=Ce^{\frac{1}{2}x}\cos(3x+\alpha)$ etc. Condone mixed variables for this mark. May not be seen until the end.

B1: Correct form for the particular integral selected. Must include ke^{5x} but accept with any extra terms that will disappear when constants found.

M1: Differentiates their PI twice and substitutes into the equation. Allow sign/coefficient errors only.

M1: Proceeds to find the value of the constant(s) from a valid PI. Note that the constant value need not be simplified, so accept exact equivalents such as $\frac{6}{117}$ etc

A1ft: Correct general solution. **Must have *y* = … and be in terms of *x only*. Follow through their CF in terms of *x* only but the PI must be correct.** May appear at the start of (b) instead of the end of (a) which is fine.

(b)

M1: Uses the initial condition for x to find the value for "A" (their cosine coefficient). If by error of CF they obtain an equation in A and B then score for the attempt of producing a suitable equation.

M1: Differentiates the GS. Allow sign/coefficient errors only, so a full method for product rule.

A1ft: Correct differentiation following through their PI- but must have been of the correct form. Also allow ft for an incorrect value of A if found before differentiation

M1: Uses the initial conditions to find values for all constants in their equation.

A1: Correct particular solution. Allow exact equivalents for the coefficients. Accept without 'y = …' so e.g. PS = is ok here. **Must be in terms of *x only* with no mixed variables etc**

NOTE: Alt method – it is possible for candidates to form two simultaneous equations in A and B from the first and second derivatives and solve to find A and B. If you see this then please send to review. Any attempts at differentiating the complex number form for the CF can score full marks, and leads to

$$(y =)\ \left(-\frac{1}{39}+\frac{1}{26}i\right)e^{\left(\frac{1}{2}+3i\right)x}+\left(-\frac{1}{39}-\frac{1}{26}i\right)e^{\left(\frac{1}{2}-3i\right)x}+\frac{2}{39}e^{5x}$$

Which is mathematically equivalent to the $\frac{2}{39}e^{5x}+e^{\frac{1}{2}x}\left(-\frac{2}{39}cos\,3\,x-\frac{1}{13}sin\,3\,x\right)$

This is marked as per the main scheme