Question	Scheme	Marks
8(a)	Real line is $y = 0$, so have $u + \mathrm{i}v = \dfrac{(\sqrt{3} - \mathrm{i})(x-2)}{x+2} = \dfrac{\sqrt{3}(x-2)}{x+2} - \dfrac{(x-2)}{x+2}\mathrm{i}$	M1
	$\Rightarrow \dfrac{u}{v} = \dfrac{\sqrt{3}}{-1} \Rightarrow v = -\dfrac{1}{\sqrt{3}}u$ *	M1A1*
		(3)
	For any attempts at solutions which use image points under the transformation T for parts (a) or (b) please send to review	
(a) Alt (i)	$w = \dfrac{(\sqrt{3} - i)(z-2)}{z+2} \Rightarrow wz + 2w = \sqrt{3}z - 2\sqrt{3} - iz + 2i$ $\Rightarrow z = \dfrac{2(i - w - \sqrt{3})}{i + w - \sqrt{3}} = \dfrac{-2(u+\sqrt{3}) - 2(v-1)i}{(u-\sqrt{3}) + (v+1)i} \times \dfrac{(u-\sqrt{3}) - (v+1)i}{(u-\sqrt{3}) - (v+1)i}$	M1
	$= \dfrac{-2(u^2-3) + 2(v+1)(u+\sqrt{3})i - 2i(uv - \sqrt{3}v - u + \sqrt{3}) - 2(v^2-1)}{(u-\sqrt{3})^2 + (v+1)^2}$ $y = 0 \Rightarrow 2(v+1)(u+\sqrt{3}) - 2(uv - \sqrt{3}v - u + \sqrt{3}) = 0$	M1
	$2(uv + \sqrt{3}v + u + \sqrt{3}) - 2uv + 2\sqrt{3}v + 2u - 2\sqrt{3} = 0$ $\Rightarrow 4\sqrt{3}v + 4u = 0 \Rightarrow v = -\dfrac{1}{\sqrt{3}}u *$	A1*
Alt (ii)	$w = \dfrac{(\sqrt{3} - i)(z-2)}{z+2} \Rightarrow ux + 2u + ivx + 2iv = \sqrt{3}x - 2\sqrt{3} - ix + 2i$ $\Rightarrow ux + 2u = \sqrt{3}x - 2\sqrt{3}$ and $vx + 2v = -x + 2$	M1
	$\Rightarrow x = \frac{-2u-2\sqrt{3}}{u-\sqrt{3}}$ and $x = \frac{-2v+2}{v+1}$ $\Rightarrow \dfrac{-2u - 2\sqrt{3}}{u - \sqrt{3}} = \dfrac{-2v + 2}{v + 1}$	M1
	$\Rightarrow -2uv - 2u - 2\sqrt{3}v - 2\sqrt{3} = -2uv + 2\sqrt{3}v + 2u - 2\sqrt{3}$ $4\sqrt{3}v = -4u \Rightarrow v = -\frac{1}{\sqrt{3}}u*$	A1*

	Note: If a candidate finds an expression for z in (a) then this may be used/seen in (b) and can score M1A1 in part (b) <u>if used in (b)</u>	
(b)	$w=\dfrac{\left(\sqrt{3}-\mathrm{i}\right)(z-2)}{z+2}\Rightarrow w(z+2)=\left(\sqrt{3}-\mathrm{i}\right)z-2\left(\sqrt{3}-\mathrm{i}\right)\Rightarrow z=\ldots \text{ or } kz=\ldots$	**M1**
	$z=\dfrac{-2w-2\left(\sqrt{3}-\mathrm{i}\right)}{w-\sqrt{3}+\mathrm{i}}$ or $z\left(w-\sqrt{3}+\mathrm{i}\right)=-2w-2\left(\sqrt{3}-\mathrm{i}\right)$	**A1**
	$\lvert z\rvert=2\Rightarrow 2\lvert u+\mathrm{i}v-\sqrt{3}+\mathrm{i}\rvert=\lvert -2u-2\mathrm{i}v-2\sqrt{3}+2\mathrm{i}\rvert$	**M1**
	$\Rightarrow 4\left(u-\sqrt{3}\right)^2+4(v+1)^2=\left(2u+2\sqrt{3}\right)^2+(2v-2)^2$ $\Rightarrow \cancel{4u^2}-8\sqrt{3}u+12+\cancel{4v^2}+8v+4=\cancel{4u^2}+8\sqrt{3}u+12+\cancel{4v^2}-8v+4$	**M1**
	$\Rightarrow 16v=16\sqrt{3}u\Rightarrow v=\sqrt{3}u$ o.e.	**A1**
		(5)
(b) **Alt (i)**	$w=\dfrac{\left(\sqrt{3}-\mathrm{i}\right)(z-2)}{z+2}\Rightarrow w(z+2)=\left(\sqrt{3}-\mathrm{i}\right)z-2\left(\sqrt{3}-\mathrm{i}\right)\Rightarrow z=\ldots \text{ or } kz=\ldots$	**M1**
	$z=\dfrac{-2w-2\left(\sqrt{3}-\mathrm{i}\right)}{w-\sqrt{3}+\mathrm{i}}$ or $z\left(w-\sqrt{3}+\mathrm{i}\right)=-2w-2\left(\sqrt{3}-\mathrm{i}\right)$	**A1**
	$\lvert z\rvert=2\Rightarrow \lvert w-\sqrt{3}+i\rvert=\lvert -w-\sqrt{3}+i\rvert$ $\lvert w-(\ldots)\rvert=\lvert w-(\ldots)\rvert$	**M1**
	$\lvert w-(\sqrt{3}-i)\rvert=\lvert w-(-\sqrt{3}+i)\rvert$ So perp bisector of $(\sqrt{3},-1)$ and $(-\sqrt{3},1)\Rightarrow v=\sqrt{3}u$ o.e.	**M1A1**
(c)	$R=\{z\in\mathbb{C}:\lvert z\rvert<2\}\cap\{z\in\mathbb{C}:Im\,z>0\}$	
	$v=\sqrt{3}u\Rightarrow arg\,w=arctan\sqrt{3}=\ldots\left(\dfrac{\pi}{3}\right)$	**M1**
	$v=-\dfrac{1}{\sqrt{3}}u\Rightarrow arg\,w=arctan-\dfrac{1}{\sqrt{3}};=-\dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$	**M1A1**
	$z=0\Rightarrow w=\mathrm{i}-\sqrt{3}$ so need $\beta=\dfrac{5\pi}{6}$	**M1**
	Image region is $\left\{w\in\mathbb{C}:\frac{\pi}{3}<arg\,w<\frac{5\pi}{6}\right\}$	**A1***
		(5)
		(13 marks)

Notes:

(a)

M1: Realises real line is $y = 0$ and substitutes into the equation for w

M1: Equates real and imaginary parts and solves simultaneously to eliminate x or equivalent work

A1*: Correct work leading to the given equation, no errors seen. At least one intermediate step required after identifying- which may be implied- the real and imaginary parts (u and v in terms of x)

Alt (i):

M1: Makes z the subject and multiplies by a valid conjugate. Accept any suitable attempt to make z the subject

M1: Extracts the imaginary part and sets equal to 0 to achieve an unsimplified line equation in u and v

A1*: Achieves the correct line equation with no errors seen

Alt (ii)

M1: Replaces $w = u + iv$ and $z = x$, then rearranges the given transformation and equates the real and imaginary parts

M1: Finds two expressions for x and equates

A1*: Achieves the correct equation with no errors seen

(b)

M1: Rearranges the transformation to make z the subject, or at least as far as isolating all the z terms together.

A1: Correct expression for z or with z isolated within the expression.

M1: Applies the circle equation and substitutes $w = u + iv$

M1: Expands the moduli and cancels the square terms to arrive at an equation for a line.

A1: Correct equation (any equivalent form acceptable)

(b) Alt (i)

M1: Rearranges the transformation to make z the subject, or at least as far as isolating all the z terms together.

A1: Correct expression for z or with z isolated within the expression.

M1: Processes to an expression of the form $|z - a| = |z - b|$

M1: Recognises that the locus is a perpendicular bisector of $(\sqrt{3}, -1)$ and $(-\sqrt{3}, 1)$

A1: Achieves the correct equation

(c)

M1: Attempts the argument for the line from the image of the circle from their line equation from part (b).

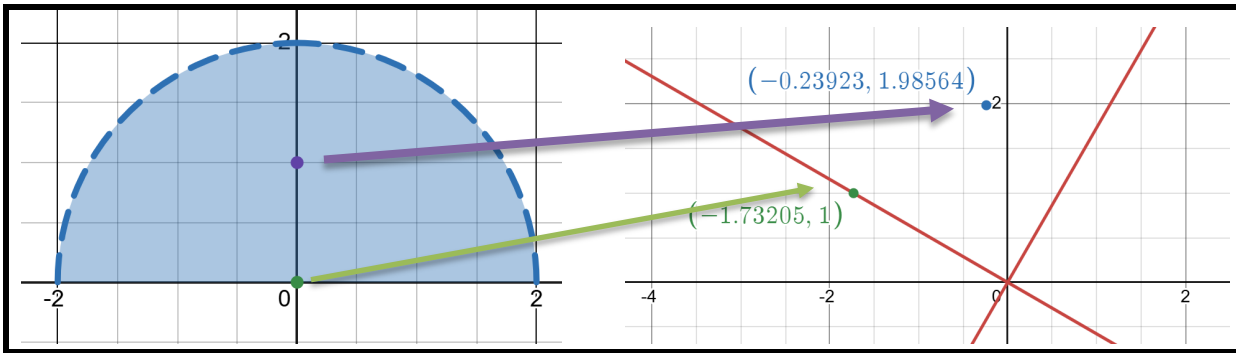
M1: Attempts the argument for the image of the real axis as given in (a)

A1: Correct possible arguments for the image of real axis identified. Need not select at this stage, so accept if just $-\frac{\pi}{6}$ is given.

M1: Uses a point on the real line segment of the boundary, or in the region R or elsewhere to determine which arguments are needed. May see image of i used (which is $\frac{4-3\sqrt{3}}{5}+\frac{3+4\sqrt{3}}{5}$i) to identify the correct quadrant, which is fine.

Alt: Uses the condition that Im z > 0 in their imaginary part for z to **show that** $v > -\frac{1}{\sqrt{3}}u$ and $|z| < 2$ in their expression for z to **show that** $v > \sqrt{3}u$

A1: Correct answer obtained with full justification. Condone the omission of $w \in \mathbb{C}$ etc



In (c) candidates are effectively finding which side of the lines found in (a), (b) the points inside the semicircle with radius 2 and positive imaginary part, are mapped to under T.

The origin is mapped to point $\left(-\sqrt{3}, 1\right)$ which is $(-1.73\ldots, 1)$ shown above, which is on the half line with argument $\frac{5\pi}{6}$

The point i which is $(0, 1)$ is mapped to $\frac{4-3\sqrt{3}}{5}+\frac{3+4\sqrt{3}}{5}\mathrm{i}$ which is $(-0.239.., 1.985..)$ shown above

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom