8. A transformation T from the z-plane, where $z = x + \mathrm{i}y$ to the w-plane, where $w = u + \mathrm{i}v$, is given by

$$w = \frac{\left(\sqrt{3} - \mathrm{i}\right)(z - 2)}{z + 2} \qquad z \neq -2$$

(a) Show that the real axis in the z-plane is mapped by T onto the line with equation $v = -\frac{1}{\sqrt{3}}u$ in the w-plane.

(3)

(b) Show that the circle in the z-plane with equation $|z| = 2$ is mapped by T onto a line in the w-plane, stating clearly an equation for this line.

(5)

The region R in the z-plane is defined by

$$\{z \in \mathbb{C} : |z| < 2\} \cap \{z \in \mathbb{C} : \mathrm{Im}\, z > 0\}$$

(c) Determine the image of R under T, giving your answer in the form

$$\{w \in \mathbb{C} : \alpha < \arg w < \beta\}$$

where α and β are rational multiples of π

(5)

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(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

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