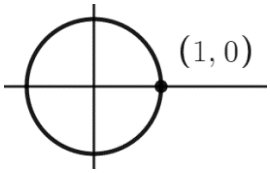
Question Number	Scheme	Notes	Marks
3(a)		Circle centred at origin with radius of 1 correctly indicated in any way. May use $\pm i$ or ± 1 in appropriate position on imaginary axis.	B1
			(1)
(b)	$w=\dfrac{9iz-i}{z+1}$ $\Rightarrow wz+w=9iz-i \Rightarrow z(w-9i)=-w-i$ $\Rightarrow z=\dfrac{-w-i}{w-9i}=\dfrac{w+i}{9i-w}$ oe	M1: Makes z the subject A1: Correct expression oe	M1 A1
	$\lvert z\rvert=\left\lvert\dfrac{-w-i}{w-9i}\right\rvert \Rightarrow \lvert w-9i\rvert=\lvert w-(-i)\rvert$	Uses $\lvert z\rvert=1$ and obtains $\lvert\pm w\pm z_1\rvert=\lvert\pm w\pm z_2\rvert$ **Requires previous M mark.**	**d**M1
	$\Rightarrow v=$"4" or $u^2+(v-9)^2=u^2+(v+1)^2$ $-18v+81=2v+1$ $20v=80$ $v=4$	Obtains $v=k$ with k correct for their $\lvert w-pi\rvert=\lvert w-qi\rvert$ oe or substitutes $u+iv$ for w, applies Pythagoras correctly and obtains a value for v Allow "y = ..." for this mark. Condone $v = 4i$ for this mark. **Requires both previous M marks.**	**dd**M1
	$v=4$ only	Correct equation	A1
			(5)
			Total 6
Alt 1: using points on $\lvert z\rvert=1$	$z=1 \Rightarrow w=\dfrac{9i-i}{1+1}$, $z=i \Rightarrow w=\dfrac{9i^2-i}{i+1}$	M1: Uses two correct values of z in the formula to obtain two expressions for w A1: Both expressions for w correct	M1 A1
	$w=4i$, $w=\dfrac{-i-9}{1+i}\times\dfrac{1-i}{1-i}=\dfrac{-10+8i}{2}=-5+4i$	Simplifies expressions to $a + bi$ form including correct method to rationalise denominators. **Requires previous M mark.**	**d**M1
	$\Rightarrow v=$"4"	Obtains $v=b$ with b correct for their $a + bi$ forms. Allow "y = ..." for this mark. Condone $v = 4i$ for this mark. **Requires both previous M marks.**	**dd**M1
	$v=4$ only	Correct equation	A1
			(5)
Alt 2: using $u+iv$	$w=\dfrac{9iz-i}{z+1}$ $\Rightarrow wz+w=9iz-i \Rightarrow z(w-9i)=-w-i$	M1: Makes z the subject A1: Correct expression oe	M1 A1

	$\Rightarrow z=\dfrac{-w-\mathrm{i}}{w-9\mathrm{i}}=\dfrac{w+\mathrm{i}}{9\mathrm{i}-w}$ oe		
	$\Rightarrow z=\dfrac{-u-\mathrm{i}v-\mathrm{i}}{u+\mathrm{i}v-9\mathrm{i}}\times\dfrac{u-(v-9)\mathrm{i}}{u-(v-9)\mathrm{i}}$ $=\dfrac{-u^2-(v+1)(v-9)-10u\mathrm{i}}{u^2+(v-9)^2}$ $\Rightarrow x=-\dfrac{u^2+(v+1)(v-9)}{u^2+(v-9)^2}\quad y=-\dfrac{10u}{u^2+(v-9)^2}$	Replaces w with $u+v\mathrm{i}$ and rationalises the denominator and extracts real and imaginary parts	**d**M1
	$x^2+y^2=1\Rightarrow\dfrac{\left(u^2+(v+1)(v-9)\right)^2+(10u)^2}{\left(u^2+(v-9)^2\right)^2}=1$ $\Rightarrow\ldots\Rightarrow(v-4)\left(u^2+v^2-18v+81\right)=0$	Applies $x^2+y^2=1$ and simplifies to identify a factor $v-k=0$ Same conditions as main scheme.	**dd**M1
	$v=4$ only	Correct equation	A1
			(5)
Alt 3: using *x*+i*y*	$w=\dfrac{9\mathrm{i}(x+\mathrm{i}y)-\mathrm{i}}{x+\mathrm{i}y+1}\times\dfrac{x+1-\mathrm{i}y}{x+1-\mathrm{i}y}=\ldots$ $=\dfrac{-9y(x+1)+(9x-1)y+\left((9x-1)(x+1)+9y^2\right)\mathrm{i}}{(x+1)^2+y^2}$	M1: Replaces z by $x+\mathrm{i}y$ and rationalises the denominator. A1: Correct expression oe	M1 A1
	$=\dfrac{-10y+\left(9x^2+8x-1+9y^2\right)\mathrm{i}}{1+2x+x^2+y^2}$ $=\dfrac{-10y+(8x-1+9)\mathrm{i}}{1+2x+1}\left[=\dfrac{-5y+4(x+1)\mathrm{i}}{x+1}\right]$	Expands and applies $x^2+y^2=1$ to simplify to linear coefficients	**d**M1
	$=\dfrac{-5y}{x+1}+4\mathrm{i}\ \Rightarrow v=4$	Simplifies to a constant imaginary term and deduces the equation. Same conditions as main scheme.	**dd**M1
	$v=4$ only	Correct equation	A1
			(5)

Additional Notes

(a)

B1: Draws the correct circle, allow tolerance with shape, look for the intent, and indicates the radius is 1. Accept ±1 on the imaginary axis or incorrect notation of coordinates as long as the radius of 1 is implied. May be drawn as the radius. Accept $r=1$ stated next to the diagram, but not $x^2+y^2=1$ without further indication of the radius being 1.

(b)

Note for the main scheme ddM1 first deduction is from using the perpendicular bisector of 9i and −i.