Question Number	Scheme	Notes	Marks
4	$4\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-11\dfrac{\mathrm{d}y}{\mathrm{d}x}-3y=78\mathrm{e}^{3x} \quad y=\dfrac{9}{2},\ \dfrac{\mathrm{d}y}{\mathrm{d}x}=0 \text{ at } x=0$		
(a)	$y=\lambda x\mathrm{e}^{3x}\Rightarrow$ $y'=\lambda\mathrm{e}^{3x}+3\lambda x\mathrm{e}^{3x},\quad y''=3\lambda\mathrm{e}^{3x}+3\lambda\mathrm{e}^{3x}+9\lambda x\mathrm{e}^{3x}$	Correct first and second derivatives (need not be simplified).	B1
	$4\left(6\lambda\mathrm{e}^{3x}+9\lambda x\mathrm{e}^{3x}\right)-11\left(\lambda\mathrm{e}^{3x}+3\lambda x\mathrm{e}^{3x}\right)-3\left(\lambda x\mathrm{e}^{3x}\right)=78\mathrm{e}^{3x}$ $\Rightarrow 13\lambda\mathrm{e}^{3x}=78\mathrm{e}^{3x}\Rightarrow\lambda=6$	M1: Substitutes their attempts at the derivatives of form $\alpha\mathrm{e}^{3x}+\beta x\mathrm{e}^{3x}$ into the differential equation and solves for λ A1: Correct value from fully correct	M1 A1cso
			(3)
(b)	$4m^2-11m-3=0\Rightarrow(4m+1)(m-3)=0$ $\Rightarrow m=-\dfrac{1}{4},\ 3$	Forms auxiliary equation and solves (usual rules)	M1
	CF: $y=A\mathrm{e}^{-\frac{1}{4}x}+B\mathrm{e}^{3x}$	For $A\mathrm{e}^{m_1x}+B\mathrm{e}^{m_2x}$ with their real and different values of m	M1
	$y=A\mathrm{e}^{-\frac{1}{4}x}+B\mathrm{e}^{3x}+\text{"6"}x\mathrm{e}^{3x}$	Correct GS ft their non-zero λ including "y = ..."	A1ft
			(3)
(c)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{1}{4}A\mathrm{e}^{-\frac{1}{4}x}+3B\mathrm{e}^{3x}+6\mathrm{e}^{3x}+18x\mathrm{e}^{3x}$ or ft	M1: Differentiates to obtain an expression of the correct form A1: Correct ft expression	M1 A1ft
	$x=0,\ y=\dfrac{9}{2}\Rightarrow\dfrac{9}{2}=A+B$ $x=0,\ \dfrac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow 0=-\dfrac{1}{4}A+3B+6$ $\Rightarrow A=\ldots,\quad B=\ldots$	Substitutes the given values into their equations and solves simultaneously for A and B	M1
	$y=6\mathrm{e}^{-\frac{1}{4}x}-\dfrac{3}{2}\mathrm{e}^{3x}+6x\mathrm{e}^{3x}$	Fully correct particular solution (including "y = ..." unless this was penalised in part (b))	A1
			(4)
			Total 10

Additional Notes

You may mark this question as a whole. Do not be concerned about part labelling as many will answer out of order, e.g. finding CF before PI.

(a) **Note: This appears on ePEN as MMA but we are marking it as BMA**

B1: Correct first and second derivatives.

M1: Full process to find the constant. The derivatives need not have been correct for this mark but must have form $\alpha\mathrm{e}^{3x}+\beta x\mathrm{e}^{3x}\ (\alpha,\beta\neq 0)$, look for the correct process.

A1cso: Correct value from fully correct work. The $x\mathrm{e}^{3x}$ must have cancelled in their equation giving only $\lambda=6$.

(b)

M1: Correct attempt to solve the auxiliary equation. If no method is shown the correct values can imply the attempt.

M1: Correct CF for their values, which must be real and distinct.

A1: Correct solution following through their value for λ. Must include the y =..

(c)

M1: Differentiates to correct form $Pe^{-\frac{x}{4}} + Qe^{3x} + Rxe^{3x} + "6"e^{3x}$ $\left(P, Q, R \neq 0\right)$, condoning at most one slip in powers. Must involve the product rule.
A1ft: Correct expression following through on their value for "6".
M1: For a full process to set up and solve simultaneous equations in *A* and *B* (or their variables) using the given values. Do not be concerned about algebraic slips, as long as two equations are set up then allow for any values following them.
A1: Fully correct, but only penalise a missing *y* = … in the first instance it would otherwise gain a mark.
Allow from solutions which gained $\lambda = 6$ in (a) even if from incorrect work.