| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **5(a)** | $\dfrac{2}{r(r+1)(r+2)}=\dfrac{A}{r}+\dfrac{B}{r+1}+\dfrac{C}{r+2}$<br>$2=A(r+1)(r+2)+Br(r+2)+Cr(r+1)$<br>$r=0:\quad 2=2A\Rightarrow A=1$<br>$r=-1:\quad 2=-B\Rightarrow B=-2$<br>$r=-2:\quad 2=2C\Rightarrow C=1$ | Any valid method to obtain at least one constant with partial fractions of the correct form. Could be implied | M1 |
| | $\dfrac{1}{r}-\dfrac{2}{r+1}+\dfrac{1}{r+2}$ | Correct partial fractions | A1 |
| | | | **(2)** |
| **(b)** | $\displaystyle\sum_{r=1}^{n}\frac{2}{r(r+1)(r+2)}=\begin{cases}\frac{1}{1}-\frac{2}{2}+\cancel{\frac{1}{3}}+\\ \frac{1}{2}-\cancel{\frac{2}{3}}+\cancel{\frac{1}{4}}+\\ \cancel{\frac{1}{3}}-\cancel{\frac{2}{4}}+\cancel{\frac{1}{5}}+\\ \ldots+\ldots\\ \cancel{\frac{1}{n-1}}-\cancel{\frac{2}{n}}+\frac{1}{n+1}+\\ \cancel{\frac{1}{n}}-\frac{2}{n+1}+\frac{1}{n+2}\end{cases}$ | Attempts at least three rows with their partial fractions of a correct form (see notes). Implied by correct expression | M1 |
| | $=1-1+\dfrac{1}{2}+\dfrac{1}{n+1}-\dfrac{2}{n+1}+\dfrac{1}{n+2}$ | A1: Correct constant terms or algebraic terms extracted<br>A1: Fully correct expression | A1 A1 |
| | $\dfrac{1}{2}-\dfrac{1}{n+1}+\dfrac{1}{n+2}=\dfrac{(n+1)(n+2)-2(n+2)+2(n+1)}{2(n+1)(n+2)}$ | Combines terms of the form $p$, $\dfrac{q}{n+b}$, and $\dfrac{r}{n+c}$ and no others into a single fraction | M1 |
| | $=\dfrac{n^2+3n+2-2n-4+2n+2}{2(n+1)(n+2)}=\dfrac{n(n+3)}{2(n+1)(n+2)}$ | Correct expression | A1 |
| | | | **(5)** |
| **(c)** | $\dfrac{n(n+3)}{2(n+1)(n+2)}>\dfrac{7}{15}$<br>$15n^2+45n>14n^2+42n+28$ | Forms inequality or equation, cross-multiplies or puts over common denominator and expands brackets | M1 |
| | $n^2+3n-28>0$<br>$(n+7)(n-4)>0$<br>$\Rightarrow n=\ldots$ or $n>\ldots\ (n>4)$ | Forms and solves 3TQ inequality or equation (usual rules) | dM1 |
| | $n=5$ | 5 only | A1 |
| | | | **(3)** |
| | | | **Total 10** |

Additional Note

(a)

M1: Must be correct partial fraction form with three fractions. Note that $\dfrac{A}{r(r+1)}+\dfrac{B}{(r+1)(r+2)}$ is not a complete partial fraction form so is M0, but this form can be used to score full marks in part (b).

A1: Correct form.

(b)

M1: For evidence of correctly applying the method of differences to the question. This may be implied by extraction of the correct terms if no/incomplete method is seen. If method is shown they should give at least three lines of fractions to show cancelling.

A1: For either the correct constant terms at the start or the correct algebraic terms at the end extracted from the differences.

A1: All correct terms extracted (and not others)

Note that $\frac{1}{r(r+1)}-\frac{1}{(r+1)(r+2)}$ will give $\frac{1}{2}-\frac{1}{6}+\frac{1}{6}-\frac{1}{12}+\ldots-\frac{1}{(n+1)(n+2)}=\frac{1}{2}-\frac{1}{(n+1)(n+2)}$ which can score M1A1A1, and then the next M for combining these.

M1: Combines the terms to a single fraction provided terms of suitable forms were extracted – as shown in scheme. The denominator must be correct but allow a slip in the numerator.

A1: Correct answer from correct work.

(c)

M1: Sets up and forms the required inequality, or accept as an equality formed, and proceeds to form an unsimplified quadratic equation or inequality by cross multiplying or by **correctly** putting terms over a common denominator and expanding the $n(n+3)$ and $(n+1)(n+2)$ brackets.

dM1: Solves the resulting 3 term quadratic (accept if just one root found). Note: answers which try to solve for the whole solution set for *n* will carry this out as part of the working, and can score full marks. Solutions to the quadratic by calculator are acceptable – you may need to check and accept if only one root of their quadratic is given.

A1: Correct answer only.

Note: Trial and Improvement or calculator methods which do not form a quadratic inequality or equation will score no marks.