6: (a) Show that the substitution $z = \frac{1}{y^2}$ transforms the differential equation

$$2\frac{dy}{dx} + (\cot x)y + (\tan x \sec x)y^3 = 0 \qquad 0 < x < \frac{\pi}{2} \qquad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - (\cot x)z = \tan x \sec x \qquad 0 < x < \frac{\pi}{2} \qquad \text{(II)}$$

(3)

(b) Hence determine the general solution of differential equation (I), giving your answer in the form $y^2 = f(x)$

(5)

Given that $y^2 = \frac{4\sqrt{3}}{3}$ when $x = \frac{\pi}{6}$

(c) determine the exact values of y when $x = \frac{\pi}{3}$

(3)

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Question 6 continued

(Total for Question 6 is 11 marks)