Question Number	Scheme	Notes	Marks
7(a)	$x=\mathrm{e}^u \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{e}^u=x$ or $u=\ln x \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{1}{x}$ $\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y}{\mathrm{d}u}\times\dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{1}{x}\dfrac{\mathrm{d}y}{\mathrm{d}u}$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y}{\mathrm{d}u}\times\mathrm{e}^{-u}$ oe	Correct form for equation in $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}u}$	**M1**
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{1}{x}\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}\dfrac{\mathrm{d}u}{\mathrm{d}x}-\dfrac{1}{x^2}\dfrac{\mathrm{d}y}{\mathrm{d}u}$ or $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\mathrm{e}^{-u}\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}\dfrac{\mathrm{d}u}{\mathrm{d}x}-\mathrm{e}^{-2u}\dfrac{\mathrm{d}y}{\mathrm{d}u}$ oe	Correct form for an equation linking $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}$	**M1**
	$2x^2\left(\dfrac{1}{x^2}\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}-\dfrac{1}{x^2}\dfrac{\mathrm{d}y}{\mathrm{d}u}\right)+3x\left(\dfrac{1}{x}\dfrac{\mathrm{d}y}{\mathrm{d}u}\right)-y=27x^2$	Substitutes to eliminate $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}u}{\mathrm{d}x}$	**M1**
	$2\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}+\dfrac{\mathrm{d}y}{\mathrm{d}u}-y=27\mathrm{e}^{2u}$ *	Fully correct proof	**A1***
			(4)
(b)	$2m^2+m-1=(2m-1)(m+1)=0\Rightarrow m=\dfrac{1}{2},\ -1$	Forms auxiliary equation and solves (usual rules)	**M1**
	$(y=)A\mathrm{e}^{\frac{1}{2}u}+B\mathrm{e}^{-u}$	Correct CF for their two real values of m	**M1**
	$y=\lambda\mathrm{e}^{2u},\ y'=2\lambda\mathrm{e}^{2u},\ y''=4\lambda\mathrm{e}^{2u}$ $\Rightarrow 8\lambda+2\lambda-\lambda=9\lambda=27\Rightarrow\lambda=3$	With correct form of PI, differentiates twice and substitutes to find a value for λ	**M1**
	$\left(y=A\mathrm{e}^{\frac{1}{2}u}+B\mathrm{e}^{-u}+3\mathrm{e}^{2u}\right)$ $y=Ax^{\frac{1}{2}}+Bx^{-1}+3x^2$	Correct GS in terms of x with no exponentials. Must include the $y=$	**A1**
			(4)
(c)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{2}Ax^{-\frac{1}{2}}-Bx^{-2}+6x$	Correct form for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ from GS of form $y=Ax^p+Bx^q+rx^2$	**M1**
	$x=\dfrac{1}{4},\ y=\dfrac{11}{16}\Rightarrow\dfrac{11}{16}=\dfrac{1}{2}A+4B+\dfrac{3}{16}$ and $x=\dfrac{1}{4},\ \dfrac{\mathrm{d}y}{\mathrm{d}x}=1\Rightarrow 1=A-16B+\dfrac{3}{2}$	Uses $x=\dfrac{1}{4}$ with $y=\dfrac{11}{16}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x}=1$ to obtain two equations in the constants (from CF)	**M1**
	$\dfrac{1}{2}A+4B=\dfrac{1}{2},\ \ A-16B=-\dfrac{1}{2}$ $\Rightarrow A=\dfrac{1}{2},\ \ B=\dfrac{1}{16}$	Having found two equations in two CF constants from using the conditions, solves for A and B. Requires previous M mark	**dM1**
	$y=\dfrac{1}{2}\left(\dfrac{1}{8}\right)^{\frac{1}{2}}+\dfrac{1}{16}\left(\dfrac{1}{8}\right)^{-1}+3\left(\dfrac{1}{8}\right)^2=\ldots$	Substitutes $x=\dfrac{1}{8}$ into their PS and obtains a value for y. Requires previous 2 M marks	**ddM1**
	$=\dfrac{\sqrt{2}}{8}+\dfrac{1}{2}+\dfrac{3}{64}=\dfrac{\sqrt{2}}{8}+\dfrac{35}{64}=\dfrac{1}{64}\left(8\sqrt{2}+35\right)$	$\dfrac{1}{64}\left(8\sqrt{2}+35\right)$ only	**A1**
			(5)

Total 13

Additional Notes:

(a)

M1: Applies the chain rule with an attempt at $\frac{du}{dx}$ or $\frac{dx}{du}$ for form an equation linking $\frac{dy}{dx}$ and $\frac{dy}{du}$

M1: Attempts the second derivative with product and chain rules to form an equation linking $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{du^2}$ of correct form.

M1: Substitutes to eliminate $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$ and $\frac{du}{dx}$ from the equation to get an equation in $\frac{d^2y}{du^2}$ and $\frac{dy}{du}$ as the only derivatives.

A1: Completely correct proof, with intermediate work and no errors.

(b)

M1: Forms and solves the auxiliary equation. For solving the auxiliary equation no method need be shown. The M can be implied by the correct values.

M1: Correct complementary function for their solutions as long as they are distinct real roots. The $y =$ is not needed and may be mislabelled as $x =$ or have x in place of u for this mark.

M1: Full process to find the particular integral, with correct form used, differentiated twice and substituted to find the value of the constant.

A1: Correct general solution in terms of x (u must be eliminated) and including the $y =$. Allow if the GS in terms of x is stated at the beginning of part (c) instead of in (b).

(c)

M1: Differentiates the general solution to the correct form.

M1: Uses the given conditions to form simultaneous equations in the constants of their equations.

dM1: Depends on previous M. Solves their two simultaneous equations (must have formed two equations from the conditions).

ddM1: Depends on previous two M's. Uses their particular solution to find the value for y when $x = \frac{1}{8}$

Condone a slip when substituting as long as it is clear they are attempting the correct x value.

A1: Correct answer only.

NB Part (c) can be done in terms of u if they do not convert back to an equation in x

M1: Differentiates the GS in terms of u : $\frac{dy}{du} = \frac{A}{2}e^{\frac{1}{2}u} - Be^{-u} + 6e^{2u}$

M1: Forms simultaneous equations using $u = \ln\frac{1}{4}$ (oe) and the value for y and $\frac{dy}{du} = x\frac{dy}{dx} = \frac{1}{4}$. So

$\frac{11}{16} = Ae^{\frac{1}{2}\ln\frac{1}{4}} + Be^{-\ln\frac{1}{4}} + 3e^{2\ln\frac{1}{4}} = \frac{A}{2} + 4B + \frac{3}{16}$ and $\frac{1}{4} = \frac{A}{2}e^{\frac{1}{2}\ln\frac{1}{4}} - Be^{-\ln\frac{1}{4}} + 6e^{2\ln\frac{1}{4}} = \frac{A}{4} - 4B + \frac{3}{8}$

dM1: Solves the equations. $\Rightarrow A = ..., B = ...$

ddM1: Substitutes $u = \ln\frac{1}{8}$ into their equation in terms u to find y.

A1: cao.

Note that they must be using attempts at u, not just x values, in such an approach.

Note: where candidates confuse u and x's accept they must be substituting in the correct things, so using $\frac{dy}{du} = 1$ is 2nd M0, but if they state $\frac{dy}{dx} =$ function of x following an attempt $\frac{dy}{du}$ first then replacing u, you can allow the second M for using $\frac{dy}{dx} = 1$ in this (the first M will be lost).