7. (a) Show that the substitution $x = e^u$, where u is a function of x, transforms the differential equation

$$2x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - y = 27x^2 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$2\frac{d^2y}{du^2} + \frac{dy}{du} - y = 27e^{2u} \qquad \text{(II)}$$

(4)

(b) By solving differential equation (II), determine the general solution of differential equation (I).

Give the answer in the form $y = f(x)$ where f is a fully simplified function.

(4)

Given that when $x = \frac{1}{4}$, $y = \frac{11}{16}$ and $\frac{dy}{dx} = 1$

(c) determine the value of y when $x = \frac{1}{8}$, giving the answer in the form $\frac{1}{64}\left(p\sqrt{2} + q\right)$ where p and q are integers.

(5)

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Question 7 continued

(Total for Question 7 is 13 marks)