Question Number	Scheme	Notes	Marks
8(a)	$z^n = \cos n\theta + \mathrm{i}\sin n\theta$ or $z^{-n} = \cos(-n\theta) + \mathrm{i}\sin(-n\theta)$ or $z^{-n} = \cos n\theta - \mathrm{i}\sin n\theta$	One correct use of de Moivre on either z^n or z^{-n} seen or implied.	**M1**
	$z^n + \frac{1}{z^n} = \cos n\theta + \mathrm{i}\sin n\theta + \cos n\theta - \mathrm{i}\sin n\theta$ $= 2\cos n\theta *$	Fully correct proof with LHS and RHS both seen and expressions explicitly added.	**A1***
	Alternative: $z^n + \frac{1}{z^n} = \mathrm{e}^{\mathrm{i}n\theta} + \mathrm{e}^{-\mathrm{i}n\theta}$ (M1) $= 2\left(\frac{1}{2}\left(\mathrm{e}^{\mathrm{i}n\theta} + \mathrm{e}^{-\mathrm{i}n\theta}\right)\right) = 2\cos n\theta$ (A1*)		
			(2)
(b)	$\left(z + \frac{1}{z}\right)^4 = z^4 + 4z^3\left(\frac{1}{z}\right) + 6z^2\left(\frac{1}{z}\right)^2 + 4z\left(\frac{1}{z}\right)^3 + \left(\frac{1}{z}\right)^4$	Expands $\left(z + \frac{1}{z}\right)^4$ with correct binomial coefficients.	**M1**
	$\left(z + \frac{1}{z}\right)^4 = z^4 + \frac{1}{z^4} + 4\left(z^2 + \frac{1}{z^2}\right) + 6$ $\Rightarrow (2\cos\theta)^4 = 2\cos 4\theta + 4(2\cos 2\theta) + 6$	Correct use of $z^n + \frac{1}{z^n} = 2\cos n\theta$ shown for both terms	**M1**
	$16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$ $\Rightarrow \cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$	M1: Achieves expression for $\cos^4\theta$ of the correct form. A1: Correct expression.	**M1** **A1**
			(4)
(c)	$\pi\int y^2\mathrm{d}x = \pi\int \cos^4 x(1+\sin x)\,\mathrm{d}x$	Uses volume of revolution formula	**M1**
	$= \{\pi\}\int(\cos^4 x + \sin x\cos^4 x)\,\mathrm{d}x$ $= \{\pi\}\int\left(\frac{1}{8}(\cos 4x + 4\cos 2x + 3) + \sin x\cos^4 x\right)\mathrm{d}x$ $= \{\pi\}\left[\frac{1}{8}\left(\frac{1}{4}\sin 4x + 2\sin 2x + 3x\right) - \frac{1}{5}\cos^5 x\right]$	M1: Uses their answer to (b) and integrates it to the correct form M1: $\ldots\sin x\cos^4 x \to \ldots\cos^5 x$ A1: Fully correct integration • See notes	**M1** **M1** **A1**
	$= \{\pi\}\left[\left(\frac{1}{8}\left(0 + 2 + \frac{3\pi}{4}\right) - \frac{1}{5}\left(\frac{\sqrt{2}}{2}\right)^5\right) - \left(\frac{1}{8}\left(0 + 0 - \frac{3\pi}{2}\right) - 0\right)\right]$	Applies limits $\frac{\pi}{4}$ and $-\frac{\pi}{2}$ appropriately to an attempt at the integral. Depends on one of the two preceding Ms being scored.	**dM1**
	$= \pi\left(\frac{1}{4} + \frac{3\pi}{32} - \frac{\sqrt{2}}{40} + \frac{3\pi}{16}\right) = \pi\left(\frac{1}{4} + \frac{9\pi}{32} - \frac{\sqrt{2}}{40}\right)$ $= \frac{\pi}{160}\left(40 + 45\pi - 4\sqrt{2}\right)$	Correct answer in correct form	**A1**
			(6)
			Total 12
			PAPER TOTAL: 75

Additional Notes:

(a)

M1: One correct use of de Moivre on either z^n or z^{-n} seen or implied. May be done separately or within working.

A1: Correctly completes the proof with no errors seen. The LHS and RHS of the identity must both be seen and expressions explicitly added and the $-\mathrm{i}\sin n\theta$ seen. Condone missing brackets in e.g $\cos -n\theta$ terms if intent is clear.

Minimum for sufficient working is $z^n + \dfrac{1}{z^n} = \cos n\theta + \mathrm{i}\sin n\theta + \cos n\theta - \mathrm{i}\sin n\theta = 2\cos n\theta$

Alternatively accept if exponential forms are used.

M1: Applies $\cos\theta + \mathrm{i}\sin\theta = \mathrm{e}^{\mathrm{i}\theta}$ (may be implied) and substitutes into the given equation, with correct index work on both terms.

A1: Correctly rearranges to make the $\cos(n\theta)$ identity clear and simplifies to the given result. The minimum requirement is all four statements seen in the MS.

(b)

M1: Attempts to expand $\left(z + z^{-1}\right)^4$. Correct binomial coefficients must be used, terms need not be simplified. Condone at most one slip in powers.

M1: Gathers the relevant terms (must see the grouping) and clearly applies $z^n + \dfrac{1}{z^n} = 2\cos n\theta$ to both terms to obtain each in $\cos n\theta$ terms. The "2" must be seen or implied by correct working.

M1: Sets their expression equal to $\left(2\cos\theta\right)^4$ and makes $\cos^4\theta$ the subject with correct form.

A1: Correct answer with no clearly incorrect steps throughout (but condone minor slips which are recovered).

Note: M1M0M1A1 is possible in this part if the groups of terms is never shown but they jump directly to the expression in $\cos n\theta$ terms.

(c) The π is only required for the first and last marks.

M1: For the correct use of the volumes of revolution formula applied to the problem, but condone a slip in squaring if the intention is clear (e.g. accept $\cos^2 x(1+\sin x)$ as the attempt to use y^2). The π must be included at some stage but may be added later. Limits not required for this mark.

M1: Applies the answer to part (b) and integrates to obtain an integral of $\cos^4\theta$ the correct form. Slips in coefficients may have been made. The π and limits are not required.

M1: Correct form for $\int \sin x\cos^4 x\,\mathrm{d}x = K\cos^5 x$ The π and limits are not required.

Note an alternative here is to use part (b) again and the product to sum rule:

$$\int \cos^4 x\sin x\,\mathrm{d}x = \frac{1}{8}\int \cos 4x\sin x + 4\cos 2x\sin x + 3\sin x\,\mathrm{d}x$$

$$= \frac{1}{8}\int \frac{1}{2}\left(\sin 5x - \sin 3x\right) + 2\left(\sin 3x - \sin x\right) + 3\sin x\,\mathrm{d}x$$

$$= \frac{1}{8}\left[\frac{1}{2}\left(-\frac{1}{5}\cos 5x + \frac{1}{3}\cos 3x\right) - \frac{2}{3}\cos 3x + \frac{1}{2}\cos x - 3\cos x\right]$$

A1: Fully correct integration of both terms. The π and limits are not required.

dM1: Depends on at least one of the two preceding Ms. Applies correct limits to the integral as long as a valid attempt at the integral has been made.

A1: Correct answer in correct form.

NB Allow SC M1M0M1A0dM1A1 for those who expand from scratch instead of using the result from (b).

<table>
<tr><td rowspan="3">Alt (b)</td><td>$\left[2\cos 4\theta\right]=\left(\cos\theta+\mathrm{i}\sin\theta\right)^4+\left(\cos\theta-\mathrm{i}\sin\theta\right)^4$
$=c^4+4c^3\mathrm{i}s+6c^2\left(\mathrm{i}s\right)^2+4c\left(\mathrm{i}s\right)^3+\left(\mathrm{i}s\right)^4$
$+c^4-4c^3\mathrm{i}s+6c^2\left(\mathrm{i}s\right)^2-4c\left(\mathrm{i}s\right)^3+\left(\mathrm{i}s\right)^4$</td><td>Uses the identity, applies De Moivre and expands at least one set of brackets with correct binomial coefficients.</td><td>M1</td></tr>
<tr><td>$\Rightarrow 2\cos 4\theta=2\cos^4\theta-12\cos^2\theta\sin^2\theta+2\sin^4\theta$</td><td>Correct use of $z^4+z^{-4}=2\cos 4\theta$ to get only real terms in sine and cosine.</td><td>M1</td></tr>
<tr><td colspan="2">$\Rightarrow\cos^4\theta=\cos 4\theta+6\left(\frac{1}{2}+\frac{1}{2}\cos 2\theta\right)\left(\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)-\left(\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)^2$
$=\cos 4\theta+\frac{3}{2}\left(1-\cos^2 2\theta\right)-\frac{1}{4}\left(1-2\cos 2\theta+\cos^2 2\theta\right)$
$=\cos 4\theta-\frac{3}{2}\left(\frac{1}{2}+\frac{1}{2}\cos 4\theta\right)+\frac{1}{2}\cos 2\theta-\frac{1}{4}\left(\frac{1}{2}+\frac{1}{2}\cos 4\theta\right)+\frac{5}{4}$
$=\frac{1}{8}\left[8\cos 4\theta-6\left(1+\cos 4\theta\right)+4\cos 2\theta-\left(1+\cos 4\theta\right)+10\right]$
$\Rightarrow\cos^4\theta=\frac{1}{8}\left(\cos 4\theta+4\cos 2\theta+3\right)$
M1: Achieves expression for $\cos^4\theta$ of the correct form
A1: Correct expression</td><td>M1
A1</td></tr>
</table>

Note the question says “hence” so answers without reference to (a) will score at most one mark (e.g. purely by De Moivre). But some attempts may use the result of (a) as shown above. In which case mark as indicated.

M1: For a correct binomial expansion applied to the problem – so e.g. sets up $\left(\cos\theta+\mathrm{i}\sin\theta\right)^4+\left(\cos\theta-\mathrm{i}\sin\theta\right)^4$ and expands at least one bracket with correct coefficients. This mark may be scored for expansion of $\left(\cos\theta+\mathrm{i}\sin\theta\right)^4$ from an approach by De Moivre using $\mathrm{Re}\left(\cos\theta+\mathrm{i}\sin\theta\right)^4$. This may come from other approaches that use a binomial expansion.

M1: Evidence of the use of both sides of part (a), cancelling imaginary terms to get a real equation in cosine and sines only. Note that they must be using part (a) for this mark to be scored.

M1: Having used part (a), uses correct trigonometric identities to eliminate the $\sin\theta$ terms and reduce the powers to single power $\cos 2\theta$ terms – ie achieves correct form for $\cos^4\theta$. There may be alternate identities used (e.g. Pythagorean before using double angle identities)

A1: Correct answer.

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