8. Given that $z = \cos\theta + \mathrm{i}\sin\theta$

(a) show that, for $n \in \mathbb{Z}$

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

(2)

(b) Hence show that

$$\cos^4\theta = \frac{1}{8}(\cos 4\theta + a\cos 2\theta + b)$$

where a and b are integers to be determined.

(4)

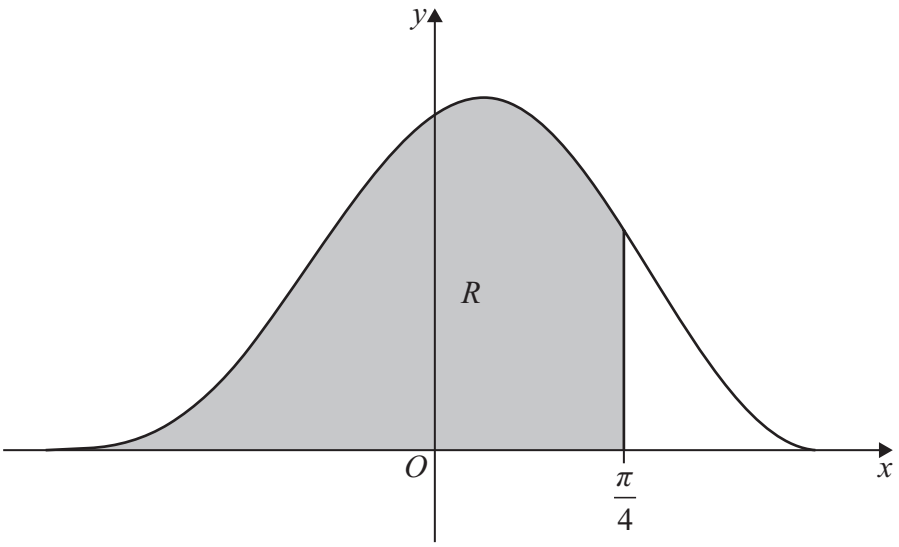


Figure 4

Figure 4 shows a sketch of the curve with equation

$$y = \cos^2 x\sqrt{1 + \sin x} \qquad -\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 4, is bounded by the curve, the x–axis and the line with equation $x = \frac{\pi}{4}$

The region R is rotated through 2π radians about the x–axis to form a solid of revolution.

(c) Use the answer to part (b) and algebraic integration to determine the exact volume of this solid.

Give the answer in the form $\frac{\pi}{160}\left(p + q\pi + r\sqrt{2}\right)$ where p, q and r are integers.

(6)

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(Total for Question 8 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

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