Question Number	Scheme	Notes	Marks
8(a)	e.g., $u = xy \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y$ or $y = ux^{-1} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = -ux^{-2} + x^{-1}\dfrac{\mathrm{d}u}{\mathrm{d}x}$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x\frac{\mathrm{d}u}{\mathrm{d}x} - u}{x^2}$ Differentiates $u = xy$ oe to obtain any correct equation in $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$		**B1**
	$\dfrac{\mathrm{d}u}{\mathrm{d}x} = x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y \Rightarrow \dfrac{\mathrm{d}^2u}{\mathrm{d}x^2} = x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + \dfrac{\mathrm{d}y}{\mathrm{d}x} + \dfrac{\mathrm{d}y}{\mathrm{d}x}$ or e.g., $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = -x^{-2}\dfrac{\mathrm{d}u}{\mathrm{d}x} + 2x^{-3}u - x^{-2}\dfrac{\mathrm{d}u}{\mathrm{d}x} + x^{-1}\dfrac{\mathrm{d}^2u}{\mathrm{d}x^2} \Rightarrow -2x^{-2}\left(x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y\right) + 2x^{-2}y + x^{-1}\dfrac{\mathrm{d}^2u}{\mathrm{d}x^2}$ Differentiates again and obtains an equation of the correct form before given answer i.e., $A\dfrac{\mathrm{d}^2u}{\mathrm{d}x^2} = Bx\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + C\dfrac{\mathrm{d}y}{\mathrm{d}x}$ oe Allow if there are clearly cancelling terms involving other variables/derivatives or the work makes completely clear how the given answer is reached		**M1**
	$\Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{1}{x}\left(\dfrac{\mathrm{d}^2u}{\mathrm{d}x^2} - 2\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)*$	Obtains the given answer with no errors. No intermediate step is required	**A1***
			(3)
(b)	$x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2(2x+1)\dfrac{\mathrm{d}y}{\mathrm{d}x} + 13xy = 17\mathrm{e}^{3x} - 4y$ $\Rightarrow x\dfrac{1}{x}\left(\dfrac{\mathrm{d}^2u}{\mathrm{d}x^2} - 2\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) + 4x\dfrac{1}{x}\left(\dfrac{\mathrm{d}u}{\mathrm{d}x} - y\right) + 2\dfrac{\mathrm{d}y}{\mathrm{d}x} + 13xy = 17\mathrm{e}^{3x} - 4y$ Substitutes the given second derivative and their first derivative into DE (I)		**M1**
	$\Rightarrow \dfrac{\mathrm{d}^2u}{\mathrm{d}x^2} + 4\dfrac{\mathrm{d}u}{\mathrm{d}x} - 4y + 13u = 17\mathrm{e}^{3x} - 4y$ $\Rightarrow \dfrac{\mathrm{d}^2u}{\mathrm{d}x^2} + 4\dfrac{\mathrm{d}u}{\mathrm{d}x} + 13u = 17\mathrm{e}^{3x}*$	Obtains the given answer with an intermediate step after substitution and no errors	**A1***
			(2)
(c)	$m^2 + 4m + 13 = 0 \Rightarrow m = -2 \pm 3\mathrm{i}$	Solves $m^2 + 4m + 13 = 0$ (allow miscopy and apply usual rules if necessary but may use calculator but if so must get a correct root for their 3TQ)	**M1**
	$\{u =\}\, \mathrm{e}^{-2x}(A\cos 3x + B\sin 3x)$	Correct CF	**A1**
	Note other CFs are possible: $\{u =\}\, A\mathrm{e}^{(-2+3\mathrm{i})x} + B\mathrm{e}^{(-2-3\mathrm{i})x}$, $A\mathrm{e}^{-2x}\cos(3x + B)$		
	$u = \lambda\mathrm{e}^{3x} \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = 3\lambda\mathrm{e}^{3x} \Rightarrow \dfrac{\mathrm{d}^2u}{\mathrm{d}x^2} = 9\lambda\mathrm{e}^{3x}$ $\Rightarrow \dfrac{\mathrm{d}^2u}{\mathrm{d}x^2} + 4\dfrac{\mathrm{d}u}{\mathrm{d}x} + 13u = (9\lambda + 12\lambda + 13\lambda)\mathrm{e}^{3x}$	Starts with the correct PI form, differentiates twice (obtaining correct (changed) forms) and substitutes into LHS	**M1**
	$\Rightarrow 9\lambda + 12\lambda + 13\lambda = 17 \Rightarrow \lambda = ...$	Solves to find the unknown in the PI	**dM1**
	$u = \mathrm{e}^{-2x}(A\cos 3x + B\sin 3x) + \dfrac{1}{2}\mathrm{e}^{3x}$	Correct GS of equation (II). Must be $u = ...$	**A1**
			(5)
(d)	$y = \dfrac{1}{x}\left(\mathrm{e}^{-2x}(A\cos 3x + B\sin 3x) + \dfrac{1}{2}\mathrm{e}^{3x}\right)$	Correct follow through equation. Allow with their CF + PI provided both are functions of x (allow e.g., CF with no constant, PI with constant) Must be $y = ...$	**B1ft**
			(1)
			Total 11