8. (a) Given that $u = xy$, where u is a function of x ($x > 0$), show that

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{1}{x}\left(\frac{\mathrm{d}^2u}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x}\right)$$

(3)

(b) Hence show that the transformation $u = xy$ transforms the differential equation

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2(2x+1)\frac{\mathrm{d}y}{\mathrm{d}x} + 13xy = 17\mathrm{e}^{3x} - 4y \qquad \text{(I)}$$

into the differential equation

$$\frac{\mathrm{d}^2u}{\mathrm{d}x^2} + 4\frac{\mathrm{d}u}{\mathrm{d}x} + 13u = 17\mathrm{e}^{3x} \qquad \text{(II)}$$

(2)

(c) Solve differential equation (II) to determine a general solution for u in terms of x.

(5)

(d) Hence determine the general solution of differential equation (I).

(1)

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Question 8 continued

(Total for Question 8 is 11 marks)