Question Number	Scheme	Marks
3. (a)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}+(\tan x)y=2\cos^2 x\sin x-3\sec x$	M1
	Integrating Factor: $I=\mathrm{e}^{\int \tan x\,\mathrm{d}x}$	dM1
	$I=\sec x$	A1
	$y\sec x=\int \sec x\left(2\cos^2 x\sin x-3\sec x\right)(\mathrm{d}x)$ or $\dfrac{\mathrm{d}}{\mathrm{d}x}(y\sec x)=\sec x\left(2\cos^2 x\sin x-3\sec x\right)$	M1
	$\int 2\cos x\sin x\,\mathrm{d}x=\sin^2 x$ or $-\cos^2 x$ or $-\dfrac{1}{2}\cos 2x$ **Must follow the previous method mark**	A1
	$\int -3\sec^2 x\,\mathrm{d}x=-3\tan x$ **Must follow the previous method mark**	A1
	Follow through their integration and their integrating factor: i.e. $y=\text{their}\left(\sin^2 x-3\tan x+k\right)\div\text{their}\sec x$ Examples of a correct answer: $y=\cos x\sin^2 x-3\sin x+k\cos x$ or $y=-\dfrac{1}{2}\cos x\cos 2x-3\sin x+k\cos x$ or $y=-\cos^3 x-3\sin x+k\cos x$	A1ft
		(7)

Notes

M1: Attempt to divide through by cos *x*. If the intention is not clear must see at least 2 terms divided by cos *x*. May be implied by a correct integrating factor.

dM1: $I=\mathrm{e}^{\int \pm\text{their P}(x)\,(\mathrm{d}x)}$ from $\dfrac{\mathrm{d}y}{\mathrm{d}x}+Py=\ldots$

Dependent on the first method mark. May be implied by use of sec *x* oe as the integrating factor.

A1: $\dfrac{1}{\cos x}$ or $(\cos x)^{-1}$ or $\sec x$ and condone $\cos^{-1} x$ if the intention is clear.

M1: Attempts to use their integrating factor correctly e.g.

$$y\times\text{their IF}=\int Q(x)\times\text{their IF}(\mathrm{d}x) \text{ or } \frac{\mathrm{d}}{\mathrm{d}x}(y\times\text{their IF})=Q(x)\times\text{their IF}$$

from their $\dfrac{\mathrm{d}y}{\mathrm{d}x}+P(x)y=Q(x)$

If there is any doubt, must multiply at least one of their $Q(x)$ terms by their IF

A1: Correct integration of $2\cos x\sin x$. **Must follow the previous method mark.**

A1: Correct integration of $-3\sec^2 x$. **Must follow the previous method mark.**

A1ft: Follow through their integration (must be a changed function and not just and their $Q(x)\times\text{their IF}$) and their integrating factor but must be *y* = … with the constant dealt with correctly. **Depends on the third method mark.**

Allow any equivalent correct or correct ft expressions which may be unsimplified so allow e.g. $-3\dfrac{\tan x}{\sec x}$ for $-3\sin x$

(b)	e.g. $3\sqrt{3} = -\frac{1}{2}\cos\frac{\pi}{3}\cos 2\frac{\pi}{3} - 3\sin\frac{\pi}{3} + k\cos\frac{\pi}{3} \Rightarrow k = ...$	M1
	$y = \cos x \sin^2 x - 3\sin x + k\cos x$ $\Rightarrow 3\sqrt{3} = \frac{1}{2}.\left(\frac{\sqrt{3}}{2}\right)^2 - 3\frac{\sqrt{3}}{2} + k\frac{1}{2} \Rightarrow k = \left(9\sqrt{3} - \frac{3}{4}\right)$ oe $y = -\frac{1}{2}\cos x \cos 2x - 3\sin x + k\cos x$ $\Rightarrow 3\sqrt{3} = -\frac{1}{2}.\frac{1}{2}\left(-\frac{1}{2}\right) - 3\frac{\sqrt{3}}{2} + k\frac{1}{2} \Rightarrow k = \left(9\sqrt{3} - \frac{1}{4}\right)$ oe $y = -\cos^3 x - 3\sin x + k\cos x$ $\Rightarrow 3\sqrt{3} = -\left(\frac{1}{2}\right)^3 - 3\frac{\sqrt{3}}{2} + k\frac{1}{2} \Rightarrow k = \left(9\sqrt{3} + \frac{1}{4}\right)$ oe	A1
	$y = \cos x \sin^2 x - 3\sin x + \left(9\sqrt{3} - \frac{3}{4}\right)\cos x$ oe or $y = -\frac{1}{2}\cos x \cos 2x - 3\sin x + \left(9\sqrt{3} - \frac{1}{4}\right)\cos x$ oe or $y = -\cos^3 x - 3\sin x + \left(9\sqrt{3} + \frac{1}{4}\right)\cos x$ oe	A1
		(3)
		Total 10

Notes

M1: Substitutes the given conditions into their $y = f(x)$ and attempts to find a value for their constant.

A1: **Correct constant** for their method from a **correct function** in part (a).

A1: Correct equation or equivalent correct equation but must be $y = \ldots$

4. (a) Express $\dfrac{4r+2}{r(r+1)(r+2)}$ in partial fractions.

(3)

(b) Hence, using the method of differences, prove that

$$\sum_{r=1}^{n} \frac{4r+2}{r(r+1)(r+2)} = \frac{n(an+b)}{2(n+1)(n+2)}$$

where a and b are constants to be found.

(5)

(Total for Question 4 is 8 marks)